## THE EXAMINATION SYSTEM IN CHINA: THE CASE OF ZHONGKAO MATHEMATICS

WU Yingkang

East China Normal University, Shanghai, China

ykwu@math.ecnu.edu.cn

Examination is a critical issue in education system in China. Zhongkao is a kind of graduation examination of junior high school, and at the same time, the entrance examination to senior high school. This paper describes the structure, features and changes in zhongkao mathematics in China based on a detailed analysis of 48 selected zhongkao mathematics papers from eight regions in recent six years. Examples of examination items are given to illustrate the identified features and changes. Zhongkao Mathematics, examination, features, changes, junior high school graduates

## INTRODUCTION

China is the birthplace of examination system. The imperial examination was started in 597 during the Sui Dynasty, and was banned in 1905 during the Qing dynasty (Li & Dai, 2009; Zhang, 1996). It lasted for about 1300 years. With the influence of the long existence of the imperial examination system, examination is of great importance in China. It attracts attention from parents, educators, teachers, students, policy makers and so on. It is a big issue in education.

There are two significant examinations for students in school education in China, which are called "zhongkao" and "gaokao". Figure 1 shows the school education system in China. Students start their nine-year compulsory education usually at six years old. Most of them stay at elementary school for six years, and junior high school for three years. In some districts like Shanghai, students stay at elementary schools for five years and junior high school for four years. At the end of Grade Nine, all students take zhongkao, which is summative assessment of the nine-year compulsory education, and more importantly, the entrance examination to senior high school. Nearly 90% of junior high school graduates continue their study. About half of them go to senior high schools, and the other half enter secondary vocational schools (Ministry of education of China, 2010a). The results of zhongkao decide whether students go to key senior high school, ordinary senior high school or vocational school. At the end of three-year senior high school study, students take gaokao, which is the entrance examination to universities. About 80% senior high school graduates are promoted to tertiary education (Ministry of education of China, 2010a). The results of gaokao decide whether senior high school graduates go to key university, ordinary university, college, or other high education institutes.

18-	University, college, high vocational schools, employment							
	Gaokao							
15-17	15-17 Senior high school, secondary vocational school							
	Zhongkao							
6-15	13-15 (12-15)	Junior high school						
	6-12 (6-11) Elementary school							
	Nine-year compulsory education							

Figure 1. School education system in China

China is a developing country. High quality education resources are still lacking. In order to be promoted into key senior high schools and key universities, Chinese students have to spend a lot of time and effort to prepare for zhongkao and gaokao. Many teachers tend to teach what are examined in zhongkao and gaokao only. Parents buy a plenty of exercise books for their kids to practice. In some districts, the three-year senior high school course is reduced to be taught in two years, and teachers spend a whole year to help students review the content of the examined subjects in gaokao. Entering good high schools or universities becomes the most important goal for many students' learning.

In order to change the above examination-oriented phenomenon, the ministry of education of China makes efforts, including making reforms on zhongkao and gaokao. The present paper focuses on zhongkao mathematics. It intends to depict a comprehensive picture of zhongkao mathematics, discussing its features and changes in light of a detailed analysis of the selected zhongkao mathematics examination papers from 2006 to 2011.

# BASIC INFORMATION ABOUT ZHONGKAO AND ZHONGKAO MATHEMATICS

#### Purpose of zhongkao

In the No. 2 document issued by ministry of education of China (2005), the purpose of zhongkao is stated as to provide complete and accurate information on junior high school students' achievement in the required subjects. The result of zhongkao has two functions. One is to evaluate whether or not students have reached the standard for graduation, and the other is to select qualified students for senior high school study.

## Organization of zhongkao

Zhongkao is organized by local educational administrative department (Wang et.al, 2004). The question-setting group for zhongkao mathematics consists of three to four mathematics teachers and teaching-research staff. The question-reviewing group is made of mathematics

education specialists and senior teachers, and most of them having previous experiences of question-setting. The marking group is composed of local core teachers. They are trained strictly before marking so as to keep the marking process fair, impartial and impersonal.

## Number of students who take zhongkao

Every year there is a large number of students taking zhongkao. According to the published statistics in the website of the ministry of education of China (2010b), there are about 18 million to 20 million junior high school graduates every year from 2006 to 2010.

## Date and key subjects for zhongkao

Zhongkao normally takes place in mid-June. Chinese, Mathematics, and English are the three key subjects in zhongkao. Hence, these three subjects attract more attention from school principals, students, parents and researchers.

## Principle for question-setting in zhongkao

The No. 2 document issued by ministry of education of China (2005) indicates that question-setting should comply with subject curriculum standards, strengthen the relationship between questions and students' real life, and attach importance to students' mastery of mathematical knowledge and skills, especially their ability to apply what they have learnt to solve problems in real context. Catchy questions are not allowed in zhongkao.

## Assessment requirements stated in Mathematics curriculum standard

The curriculum reform for basic education in China was initiated in 2001. The mathematics curriculum standard of compulsory education (experimental version) was first implemented in September 2001 in 42 national level experimental districts (Ma, et. al, 2009). In 2005, the curriculum was implemented nation-wide. Therefore, in 2008 all the Grade 9 students from the whole country take zhongkao based on the new curriculum.

The mathematics curriculum standard of compulsory education (experimental version) states that the main purpose of assessment is to get comprehensive information about students' learning experiences, to promote students' learning and to improve teachers' teaching (Ministry of education of China, 2001).

The standard highlights the importance of process assessment. It says that "the assessment on mathematics learning should pay attention to students' learning results and their learning process" (Ministry of education of China, 2001, p. 2). The standard proposes portfolio assessment, interview, classroom observation and project work as new approaches to evaluate mathematics learning, attaches great importance to problem posing and problem solving abilities, and show respect to individual's difference so as to make every student to enjoy success in mathematics learning. For example, open-ended problems with various tackling approaches can be developed, so that students with different abilities can solve the problem at different levels. It is hoped that these ideas on assessment could be manifest in zhongkao mathematics.

## **RESEARCH QUESTIONS**

This paper aims to describe the characteristics of zhongkao mathematics in China in recent years. More specifically, it intends to answer the following research questions.

- 1. What is the structure of zhongkao mathematics?
- 2. What are the features of zhongkao mathematics?
- 3. What are the changes in zhongkao mathematics?

## METHODOLOGY

## Samples

A total number of 48 zhongkao mathematics papers from eight province-level regions from 2006 to 2011 were selected for analysis as shown in table 1.

Name	Category	Location	Economic zone
Shanghai	centrally-administered municipality	East China	Eastern
Hebei	province	North China	Eastern
Guangdong	province	Southern China	Eastern
Anhui	province	East China	Central
Jilin	province	Northeast China	Central
Henan	province	Central China	Central
Yunnan	province	Southwest China	Western
Xinjiang	Autonomous region	Northwest China	Western

Table 1.	Characteristics	of the eight	selected	nrovince-lev	el region
	Characteristics	of the eight	. selected	province-iev	criegion

The eight regions were chosen for three reasons. First, they represent different categories of province-level regions. China mainland has 22 provinces, five autonomous regions, and four centrally-administered municipalities. The selected regions are made of six provinces, one autonomous region, and one centrally-administered municipality. Second, they locate at different part of China as stated in the table. Third, they are from different economic zones, with three from eastern, another three from central and the other two from western zone. Eastern zone is regarded as affluent and developed area in China, followed by central zone and western zone in a descending order of economical level. The number of province-level regions in eastern, central and western economic zones is twelve, nine and ten respectively (Zhang, 2010).

Wu

Shanghai was selected for one more reason. Among 65 countries, Shanghai 15-year-old students ranked at the top in mathematics in the 2009 administration of the Program for International Student Assessment (PISA). The scores from Shanghai are by no means representative of all of China. The result from an analysis of Shanghai zhongkao mathematics paper might give some implication to Shanghai students' good performance in PISA mathematics, and on the other hand, the result from a cross-region comparison on zhongkao mathematics papers could show what other regions in China require junior high school graduates to achieve compared to Shanghai.

Zhongkao mathematics papers developed by the educational administrative departments at province level rather than at prefecture-level were selected for analysis. This is because many prefectures in some provinces usually take zhongkao mathematics paper at province level as their examination paper. Therefore, zhongkao mathematics paper at province level is more representative in terms of the number of students who take it.

Hence, the 48 zhongkao mathematics papers from eight province-level regions from 2006 to 2011 were used for analysis. The characteristics of the selected papers give some confidence to the external validity of the subsequent analyses to be reported.

## Brief on the composite difficulty model

The composite difficulty model, developed by Bao (2002a, 2002b), is used as a framework to analyze zhongkao mathematics papers. The model comprises five factors, as shown in figure 2. Each factor has different levels.



Figure 2. The composite difficulty model

The cognitive demand factor has three levels, knowing, understanding and investigating. Knowing is about memorization of mathematical facts and performing routine mathematical procedures; understanding relates to flexibly applying mathematical concepts, formulas, rules, and theories, using routine mathematical methods to obtain definite results, and justifying and arguing mathematical facts; investigating involves making mathematical conjectures and building up mathematical models, and its problem solving strategies are not readily available. Tasks at this level are usually open-ended, non-routine, and exploratory in nature. The levels of the context factor are set by the distance of the material used to students. The closest is personal life (e.g., house working, school tour), next is public life (e.g., bank interest, sales of goods), and the most distant context for students is scientific ones (e.g., physical experiment). None context items refer to those intra mathematics questions.

The levels of computation and reasoning are set based on the complexity of the operations involved. The four levels of computation factor in an increasing order of complexity are none, numerical, simple symbolic (computation involving one or two steps) and complex symbolic (computation involving one or two steps). The levels of reasoning factor were none, simple (reasoning involving one or two steps) and complex (reasoning involving three or above three steps) previously. Considering the fact that many items in zhongkao mathematics requiring reasoning more than three steps, the third level was divided into two, which were moderate (reasoning involving three or four steps) and complex (reasoning involving five or above five steps).

The levels of topic coverage indicate the number of mathematical topics in a single item. In the original model, this factor has three levels, single topic, two topics, and three or above three topics. Here the third level was divided into two, three topics and four or above four topics. It was found that many items in zhongkao mathematics papers contain more than three topics. This minor adjustment could provide more accurate information.

Table 2 gives the revised levels of each factor. The difficulty index of each factor in the composite difficulty model was estimated by the formula below (Bao, 2004).

$$d_i = \frac{\sum_{j=1}^{j} n_{ij} d_{ij}}{n}$$
  $(\sum_{j=1,2,3,4,5,\ldots,j=1,2,\cdots)$ 

where di (i = 1,2,3,4,5) corresponds to the five factors;  $d_{ij}$  represents the power index of the j'th level of the i'th factor as shown in Table 2;  $n_{ij}$  is the total number of items which belong to the j'th level of the i'th factor, and the sum of  $n_{ij}$  is n (total items). For example, it was found that in 2011 the number of items at the level of knowing, understanding and investigating are 66, 197 and 17 respectively. Hence, the number of items is 280, and the index of cognitive demand in 2011 is calculated by expression  $(66 \times 1 + 197 \times 2 + 17 \times 3) \div (66 + 197 + 17) \approx 1.83$ .

This composite difficulty level was used to compare mathematics curriculum (Bao, 2004) and mathematics examination papers (Bao, 2006). It is a useful tool in comparing mathematical tasks. It could provide important information of the five factors on zhongkao mathematics papers and its quantitative nature helps to trace the changes in zhongkao mathematics papers in recent years and across different province-level regions.

## Data coding

The selected 48 zhongkao mathematics papers were coded at the paper level and at the item level. At the paper level, information of region, year, number of questions, number of items, duration, and total marks were recorded. A question might have several items. The number of items shows the actual working load for students.

Factor	Levels	Power index
Cognitive demand	Knowing	1
	Understanding	2
	Investigating	3
Context	None	1
	Personal	2
	Public	3
	Scientific	4
Computation	None	1
	Numerical	2
	Simple symbolic (one or two steps)	3
	Complex symbolic (three or above three steps)	4
Reasoning	None	1
	Simple (one or two steps)	2
	Moderate (three or four steps)	3
	Complex (five or above five steps)	4
Topic coverage	Single topic	1
	Two topics	2
	Three topics	3
	Four or above four topics	4

Table 2: Levels under	r each factor	in the c	omposite	difficulty	model
1 doit 2. Devels under		In the e	omposite	unneurry	mouel

At the item level, information of question type (multiple choice, blank-filling, or solutionseeking), content (algebra, geometry, statistics, or synthesization), cognitive demand (knowing, understanding, investigating), context (none, personal, public, scientific), computation (none, numerical, one-step symbolic, two-step symbolic, three-step symbolic, etc.), reasoning (none, one-step, two-step, three-step, etc.), topic coverage (single, two, three, four, etc.) were coded. The codes are 1 or 0. If the item applies the category, then code 1 is given, otherwise, code 0. For solution-seeking questions, the steps of reasoning or computation required are counted based on reference answers provided by the question-setting group. In addition, mathematics topics involved for each item were listed out. For example, the item in figure 3 was coded as blank-filling (question type), geometry (content), none (context), understanding (cognitive demand), none (computation), two-step (reasoning), two topics (topic coverage), and perimeter and figure folding (mathematical topics).

As shown in the figure, the side of an equilateral triangle  $\triangle ABC$  is 1 *cm*, *D* and *E* are points on the side *AB* and *AC* respectively. Fold  $\triangle ADE$  along *DE*, and point *A* moves to *A'* which is outside  $\triangle ABC$ . The perimeter of the shaded figure is \_\_\_\_\_ *cm*.



Figure 3. An item from Hebei 2009

In the course of coding, non-traditional and non-routine questions were noted in order to explore features and changes of zhongkao mathematics papers.

## RESULTS

## Structure of zhongkao mathematics

Structure of zhongkao mathematics is discussed from two aspects, the structure of question type and the structure of content. Generally speaking, Zhongkao mathematics papers contain three types of items, which are multiple choice, blank-filling and solution-seeking questions. Multiple choice questions usually appear first, followed by blank-filling questions and solution-seeking questions in sequence. The ratio of the numbers of these three types of items is about 2:3:5 on average, showing that the number of items requiring answers only and the number of items requiring answers and solutions are about the same.

The distribution of question type across the eight regions is quite different. Four of the regions, which are Anhui, Jilin, Yunnan, Xinjiang, follow the general trend. Shanghai, Hebei, and Henan have more items requiring answers only (63%, 61% and 56%, respectively), while Guangdong has more items requiring answers and solutions (61%).

In terms of content structure, about 75% of the total items focus on merely algebra or geometry, the classical content area in school mathematics curriculum, with algebra items (39%) a bit more than geometry items (36%). Another 15% of the total items deal with statistics, a relatively new content area introduced in the mathematics curriculum standard of compulsory education in 2001. The last 10% items synthesized at least two of the three

content areas, and the most of these items relates algebra and geometry. The combination of statistics and algebra or statistics and geometry is rare.

Regarding the content distribution across regions, Shanghai, Jilin, Henan, and Yunnan follow the general trend, while the other four regions vary differently. Guangdong has more geometry items (42%) than algebra items (35%). Hebei has more synthesized items (16%), while Anhui and Xinjiang have less synthesized items (4% and 3% respectively).

Synthesized items are usually more difficult than items related to only one content area, and they normally appear as the last question(s) in zhongkao mathematics paper. There are a plenty of articles and books discussing how to tackle synthesized questions (e.g., Ma, Shu & Peng, 2011). A typical synthesized problem could be investigating the existence of points satisfying certain conditions in a graph of a function, or finding the relationship between certain variables in a varying process of a geometrical figure or figures. Figure 4 shows an example. It is the last question in 2010 Henan zhongkao mathematics paper.



Figure 4. The last question of Henan 2010

## Features of zhongkao mathematics

The features of zhongkao mathematics are summarized based on the results of data coding. The general conclusion is that on one hand zhongkao mathematics in China in recent years inherit its traditional emphasis on basic knowledge and basic skills, and on the other hand it takes on a new look which reflects the requirement from the mathematics curriculum standard for basic education issued in 2001.

#### About three minutes assigned to each item on average

The average question number and item number of the selected 48 zhongkao mathematics papers are 24 and 35, respectively. The number of questions is in the range from 22

(Guangdong) and 28 (Jilin). It is relatively stable across the eight regions compared to the number of items. The number of items varies across regions, with Hebei having 43 items on average as the most and Anhui having 31 items as the least.

Three of the eight province-level regions, which are Henan, Shanghai and Guangdong, set the examination duration as 100 minutes, and the other five regions 120 minutes. Table 3 shows the average time assigned to each item across regions and years. It suggests that a student has about 3 minutes to answer each item in zhongkao mathematics. There is not much difference in the average time assigned to each item across years. However, it could be observed from the table that students from Yunnan and Xinjiang seem to have more time for each item and students from Hebei and Jilin have less time.

	2006	2007	2008	2009	2010	2011	On average
Shanghai	2.9	2.9	3.1	2.9	3.1	2.9	3.0
Hebei	2.6	2.7	2.8	2.9	2.9	3.0	2.8
Guangdong	2.6	3.3	3.2	3.2	2.9	3.0	3.0
Anhui	3.5	4.1	4.0	3.8	3.9	3.8	3.8
Jilin	2.9	2.9	2.9	3.1	2.9	2.9	2.9
Henan	3.4	3.2	3.3	3.2	2.9	3.0	3.2
Yunnan	3.5	3.6	3.9	4.1	3.4	3.4	3.7
Xinjiang	4.0	3.4	4.0	3.5	3.5	3.8	3.7
On average	3.1	3.2	3.4	3.3	3.2	3.2	3.2

Table 3: Average time assigned to each item (minute)

## Most items at the understanding level

Concerning cognitive demand, the percentages of items at the knowing, understanding and investigating level are 24%, 70% and 6%, respectively. This result is quite different from Bao (2006). He found that the percentages of the items, from Suzhou zhongkao mathematics papers from 1999 to 2001, at the knowing, understanding and investigating levels are 42%, 58% and 0. It could be inferred that in general the number of items relying on memorization or routine mathematical procedures is decreasing, and the number of items involving applying mathematical knowledge, making conjectures is increasing. This reflects the ideas advocated in the new mathematics curriculum.

Figure 5 gives two items taken from zhongkao mathematics papers. Although these two items both involve algebraic manipulations, one is coded as at the knowing level and the other at the

understanding level. This is because the first item requires only routine mathematical procedure while the second one requires an understanding of the concept of domain of the definition.

1. Simplify the expression 
$$\left(\frac{x^2}{x-1} - \frac{2x}{1-x}\right) \div \frac{x}{x-1}$$
, and then find its value when  $x = \sqrt{3} + 1$ . (from Xinjiang 2010)

2. Simplify the expression 
$$(1 - \frac{1}{x-1}) \div \frac{x^2 - 4x + 4}{x^2 - 1}$$
, and then choose a suitable integer x from the range  $-2 \le x \le 2$  to find the value of the expression.

(from Henan 2011)

Figure 5. Items from Xinjiang 2010 and Henan 2011

Investigating items mostly appear at the end of each type of questions. Figure 4 shows a synthesized question having three items. The last item of the question is at the investigating level. The problem-solving strategy to this item is not readily available. It requires students to analyze the meaning of the item and come up with their own method. Figure 6 gives a blank-filling item which is at investigating level. It is the last blank-filling question in Yunnan 2009 paper. This item requires students to find out the underlying law from concrete examples like  $P_1$ ,  $P_2$ ,  $P_3$  and more if needed, and then apply this law to the special case of  $P_{2009}$ .

In the plane coordinate system, the coordinates of three given points are  $A_1(1, 1)$ ,  $A_2(0, 2)$ , and  $A_3(-1, 1)$ , respectively. An electronic frog is at the origin point. At the first time, the frog jumps from the origin point to its symmetrical point  $P_1$  with  $A_1$  as the center of symmetry, the second time the frog jumps from  $P_1$  to its symmetrical point  $P_2$  with  $A_2$  as the center of symmetry, the third time the frog jumps from  $P_2$  to its symmetrical point  $P_3$  with  $A_3$  as the center of symmetry, ..., following this rule, the frog continues jumping with  $A_1$ ,  $A_2$  and  $A_3$  as the centers of symmetry. When the frog jumps the 2009 times, the coordinates of the point  $P_{2009}$  are ( \_\_\_\_\_\_, \_\_\_\_\_).

Figure 6. An item from Yunnan 2009

The eight regions vary in the levels of cognitive demand. Xinjiang, Guangdong, Anhui, Jilin, Hebei, Henan and Yunnan generally follow the global trend, while Shanghai performs quite differently. The zhongkao mathematics papers from Shanghai have about 43% items at

Wu

knowing level and 54% items at the understanding level, showing Shanghai pays more attention to basic mathematical facts and procedures.

## Most items not having real context

The majority items in zhongkao mathematics do not have real context. In general, the ratio of items not having real context to those having real context is about 2:1. Among the items having real context, almost all of them relate to personal life or public life.

Among the eight regions, Shanghai and Guangdong are the top two regions having more intra mathematics items, with the percentages of 82% and 76%, Xinjiang and Hebei are the bottom two regions, with the percentages of 53% and 58%.

## Attach high importance to computation and reasoning

Computing and reasoning abilities are strongly emphasized in mathematics curriculum in China since 1960's (Zhang & Song, 2009). This tradition has been kept till now as reflected in the zhongkao mathematics papers.

In general, about two thirds of the items in zhongkao mathematics require computation. Among those items requiring computation, 40% deals with numerical computations, 35% is about simple symbolic computations (one or two steps), and the remaining 25% is about complex symbolic computations (three and more than three steps). For example, items shown in Figure 5 require at least three steps of symbolic computation. This distribution varies across the eight regions, with Anhui and Xinjiang have bigger percentages of items not requiring computation (40% and 43% respectively), and Jilin has the biggest percentage of items requiring three or above three steps of symbolic computations (26%).

Regarding the reasoning factor, the data shows that more than three-fourths of the items need mathematical reasoning. Among these items, nearly 60% involve one or two steps of reasoning, about 25% require three or four steps of reasoning, and the remaining 15% deal with five or more steps of reasoning. Figure 7 shows a blank-filling geometrical item from Shanghai 2011. Six steps of reasoning are needed to solve this item. Among the eight regions, Xinjiang is the least demanding on complex reasoning, with only 3% of the total items requiring five or above five steps of reasoning. Oppositely, 85% of the total items in Henan require reasoning, and 17% require five or above five steps of reasoning.

In Rt $\triangle ABC$ ,  $\angle C = 90^\circ$ ,  $\angle B = 50^\circ$ , point *D* is on the side *BC*, *BD* = 2*CD*.  $\triangle ABC$  is rotated *m*(0 < m < 180) degree anti-clockwise about point *D*. If point *B* happened to be on the original sides of Rt $\triangle ABC$ , then *m* = \_\_\_\_\_.



## Figure 7. An item from Shanghai 2011

Wu

#### Highlight relations among different mathematics topics

Topic coverage factor indicates the number of topics involved in a single item. In general, about 35% of the total items relate to a single topic, and the rest 65% items deal with at least two topics. Among those items relating to more than one topic, 45% relates to two topics, 27% deal with three topics, and the remaining 28% of items relate to four or above four topics. Items in figures 3 to 7 are all examples of items involving more than one topic. However, the distribution of topic coverage varies greatly among the eight regions. If the eight regions are arranged in a descending order according to its percentages of items with single topic, Xinjiang, Shanghai and Yunnan are at the top in the list, with the percentages of 49%, 46% and 40%, and Henan and Jilin are at the bottom, with the percentages of 23% and 26%.

#### Changes in zhongkao mathematics

Changes in zhongkao mathematics are summarized from two aspects, changes after the new curriculum reform, and changes in recent years.

#### Changes after the curriculum reform

A comparison between the data reported by Bao (2006) and this study are compared to trace changes after the curriculum reform implemented since 2001. It should be noted that the data of Bao (2006) are zhongkao mathematics papers of Suzhou from 1999 to 2001. Suzhou is a city located at Jiangsu province in east China. The comparison can just give a rough estimation of the changes.

Table 4 shows the difficulty indices of the five factors between Bao (2006) and this study. The difficulty indices in Bao (2006) have been re-calculated based on the revised levels of the five factors shown in Table 2. Figure 8 shows the data in a diagram. The table and the diagram show that compared to Suzhou data, the indices of computation and topic coverage from this study decreases, the indices of cognitive demand and context increase, and the index of reasoning remains almost constant. This implies a more balanced composite difficulty level. These changes indicate that zhongkao mathematics is putting more emphasis on assessing students' abilities to solve problem in real context and to flexibly apply what they have learnt to a new situation, rather than memorization of mathematical facts and procedures only. This is also manifested by the items in zhongkao mathematics papers. Two examples are given below.

	Cognitive demand	Context	computation	Reasoning	Topic coverage
This study	1.82	1.51	2.21	2.20	2.19
Suzhou	1.58	1.27	2.60	2.26	2.57

Table 4: Comparison of composite difficulty levels between Suzhou (1999-2001) and<br/>the eight regions in this study (2006-2011)



Figure 8. Comparison of composite difficulty levels between Suzhou (1999-2001) and this study (2006-2010)

The item in Figure 9 does not explicitly give the known. Instead, it requires students to find them out from the two beautiful given pictures. Students need to study the two pictures to find the relationships among different shapes. The question in Figure 10 leads students to experience the process of problem solving, from exploring a new property by making use of basic geometrical knowledge, to generalizing the property, and to applying the property to solve a real problem. This question is not only to assess students' mathematical ability, but also to guide their mathematical learning.

The pictures below, made of ten same folding fans, are named "butterfly around flower" (picture 1) and "plum blossom" (picture 2). There is no overlap of the folding fans in both figures. The size of the five acute angles in the "plum blossom" figure is ( )



Figure 9. An item from Anhui 2006

**Exploration**: In figures 1 to 3, the area of  $\triangle ABC$  is *a*.

As shown in figure 1, extend the side *BC* to *D* to make CD=BC, and connect points *D* and *A*. If the are of  $\triangle ACD$  is  $S_I$ , then  $S_I =$  (express  $S_I$  in terms of *a*)



As shown in Figure 2, extend the side *BC* to *D* to make CD=BC, and extend the side *CA* to *E* to make AE=CA, and then connect points *D* and *E*. If the area of  $\triangle DEC$  is  $S_2$ , then  $S_2 =$ \_\_\_\_\_ (express  $S_2$  in terms of *a*). Please give reasons.

On the basis of figure 2, extend *AB* to *F* to make *BF*=*AB*, and then connect points *F* and *D*, *F* and *E* to obtain  $\triangle DEF$  as shown in figure 3. If the area of the shaded figure is  $S_3$ , then  $S_3$ =\_\_\_\_\_ (express  $S_3$  in terms of *a*)

**Discovery:** we extend the sides of  $\triangle ABC$  to double its length, and then connect the extreme points to obtain  $\triangle DEF$ . We address this process as "extend  $\triangle ABC$  once". We can find that the area of  $\triangle DEF$  is \_\_\_\_\_\_ times the area of the original triangle  $\triangle ABC$ .

**Application:** Last year flowers were planted in the ground having the shape of  $\triangle ABC$  as shown in figure 4. The area of  $\triangle ABC$  is  $10m^2$ . This year more flowers are going to be planted. We extend  $\triangle ABC$  twice, the first time from  $\triangle ABC$ to  $\triangle DEF$ , the second time from  $\triangle DEF$  to  $\triangle MGH$ . Find the area of the shaded figure in figure 4.



Figure 10. An item from Hebei 2006

## Changes in the recent years

The results shown in Table 5 and Figure 11 indicate that the difficulty indices of the five factors are generally stable across the recent years. It could be observed that the difficulty index of context is decreasing slightly and the indices of topic coverage and computation are increasing a little bit. Zhongkao is an important examination. The stability of the composite difficulty levels of zhongkao mathematics is of benefit to students' preparing for the examination.

	Cognitive demand	Context	computation	Reasoning	Topic coverage
2006	1.86	1.62	2.18	2.22	2.10
2007	1.83	1.54	2.17	2.17	2.08
2008	1.77	1.60	2.21	2.13	2.11
2009	1.84	1.48	2.26	2.24	2.28
2010	1.81	1.47	2.13	2.18	2.19
2011	1.83	1.38	2.31	2.26	2.38

 Table 5 Summary of composite difficulty level across years



Figure 11 Composite difficulty levels across years

## CONCLUSIONS

From the analysis of the selected zhongkao mathematics papers and the comparison with the data reported by Bao (2006), three conclusions could be drawn. First, zhongkao mathematics stress computation, reasoning, and relations among different mathematics topics, but not so care about applications of mathematics in real context. This actually reflects characteristics of mathematical problem solving activities in China (Bao, 2009). The tradition of mathematics education in China stress solid foundation of basic knowledge and basic skills. It is believed that creative abilities and individual development is based on a sound mastery of basic knowledge and skills.

Second, under the background of the implementation of the mathematics curriculum since 2001, zhongkao mathematics remains to keep the good tradition of emphasis on computation,

Wu

reasoning and connections among mathematical topics, and presents some new features, like more process-oriented questions and more real context questions. This leads to positive changes on mathematics classroom teaching. It is hoped that these changes could relieve students from the heavy burden of coping with the examination, and become the master of their own learning.

Third, there are obvious region differences in zhongkao mathematics as reported sporadically in the paper. Generally speaking, regions from west economic zone like Xinjiang are relatively less demanding in terms of computation, reasoning, and topic coverage factors, while regions from east and central zones like Guangdong, Jilin and Henan are more demanding in these factors. Shanghai has the lowest indices in context and cognitive demand factors, and roughly an average score in computation, reasoning and topic coverage factors. This indicates that a sound mastery of basic mathematical knowledge and skills could be a possible reason for Shanghai students' good performance in PISA mathematics 2009. It also gives some confidence that students from other regions have a good chance to perform well in an international test.

However, the region differences might result in equity issue which should be taken seriously. There are other issues in zhongkao mathematics. Calculators are allowed to be used in zhongkao mathematics in some districts like Xinjiang. How to set questions in zhongkao mathematics to fit the use of calculator is a big challenge. Moreover, the mathematics curriculum standard of compulsory education (2011 version) (Ministry of education of China, 2011) has just been published. It brings new ideas in mathematics education. For example, in addition to basic knowledge and basic skills, basic methods of thinking and basic activity experiences are proposed. Hence, the traditional two basics now become four basics. How to make zhongkao mathematics to reflect the new ideas in the standard is not an easy work. What can be predicted is that zhongkao mathematics will continue to stress good traditions of Chinese mathematics teaching and learning, and at the same time adapt to the new development of mathematics education.

#### References

- Bao, J. (2002a). Comparative study on composite difficulty of Chinese and British school mathematics curricula. (Unpublished Doctoral dissertation.) East China Normal University, Shanghai, China. (in Chinese)
- Bao, J. (2002b). Comparative study on composite difficulty of Chinese and British intended mathematics curricula. *Global Education*, *31*(9), 48-52. (in Chinese)
- Bao, J. (2004). A comparative study on composite difficulty between new and old Chinese mathematics textbooks. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 208-227). Singapore: World Scientific.
- Bao, J. (2006) . A Comparative study of Mathematics tests in China and UK. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education*, 10(1),13-31.

- Bao, J. (2009). Problem solving with Chinese characteristics. In J. Wang (Ed.), *Mathematics education in China: Tradition and reality* (pp. 106-130). Nanjing: Jiangsu education press. (in Chinese)
- Li, S.,& Dai, Q. (2009). Chinese traditional culture and mathematics education. In J. Wang (Ed.), *Mathematics education in China: tradition and reality* (pp. 1-30). Nanjing: Jiangsu education press. (in Chinese)
- Ma, X., Shu, Y., & Peng, X. (2011). *Challenging the synthesized questions in Zhongkao mathematics*. Shanghai: East China normal university press. (in Chinese).
- Ma, Y., Wang, S., Zhang, D., Liu, X.,& Guo, Y. (2009). Basic education mathematics curriculum reform in China. In J. Wang (Ed.), *Mathematics education in China: Tradition and reality* (pp. 131-175). Nanjing: Jiangsu education press. (in Chinese)
- Ministry of education of China. (2001). *The mathematics curriculum standard of full-time compulsory education (experimental version)*. Beijing: Beijing normal university press. (in Chinese)
- Ministry of education of China. (2005). The guidelines on the reform of junior high school graduation examination and senior high school enrolling system of experimental areas of basic curriculum reform. Retrieved from <a href="http://www.moe.gov.cn/">http://www.moe.gov.cn/</a> publicfiles/business/<a href="http://www.moe.gov.cn/">http://www.moe.gov.cn/</a> publicfiles/business/</a> <a href="http://www.moe.gov.cn/">http://www.moe.gov.cn/</a> publicfiles/business/
- Ministry of education of China. (2010a). *Promoting rates of graduates from schools of different levels*. Retrieved from <u>http://www.moe.edu.cn/publicfiles/business/htmlfiles/</u><u>moe/s6200/201201/129607.html</u> (in Chinese)
- Ministry of education of China. (2010b). *Information on curricula education in 2010*. Retrieved from <u>http://www.moe.edu.cn/publicfiles/business/htmlfiles/moe/s6200/</u>201201/129518.html (in Chinese)
- Ministry of education of China. (2011). *Mathematics curriculum standard of compulsory education (2011 version)*. Beijing: Beijing normal university press. Retrieved from <u>http://www.moe.gov.cn/publicfiles/business/htmlfiles/moe/moe\_711/201201/xxgk\_1292</u> <u>68.html</u> (in Chinese)
- Wang, J. et.al. (2004). Evaluation report on zhongkao mathematics in 2003. In project team, *Guidance on question-setting in zhongkao mathematics in 2004* (pp. 20-43). Nanjing: Jiangsu education press. (in Chinese)
- Zhang, D. (1996). "Coping with examination" and "examination-oriented education". *Mathematical teaching*, 6, back cover. (in Chinese)
- Zhang, D. & Song, N. (2009). *Introduction to mathematics education*. Beijing: High education press. (in Chinese)
- Zhang, Z. (2010). Evolution and Evaluation of the Chinese Economic Regions Division. Journal of Shanxi Finance and Economics University (higher education edition), 13(2), 89-92. (in Chinese)