No Common Denominator

The Preparation of Elementary Teachers in **Mathematics** by America's Education Schools

Full Report
June 2008

NATIONAL COUNCIL ON TEACHER QUALITY

The executive summary of *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools* is available online from www.nctq.org.

AUTHORS:

Julie Greenberg and Kate Walsh

OUR THANKS TO:

Research analysts: Emmanuel Caudillo, Aileen Corso, Elizabeth McCorry, Stephanie Parry, Felicity Messner Ross, Michael Savoy, and Nate Sheely

Database design and technical support: Jeff Hale

Graphic design: Colleen Hale

Mathematics Advisory Group: Richard Askey, Andrew Chen, Mikhail Goldenberg, Roger Howe, Jason Kamras, James Milgram, Robin Ramos, and Yoram Sagher

Consultation: Francis (Skip) Fennell and Mark Thames

WITH PRINCIPAL FUNDING FROM:

The Brookhill Foundation, The Louis Calder Foundation, Ewing and Marion Kauffman Foundation, Exxon Mobile Foundation, and Searle Freedom Trust

Preface

The nation's higher goals for student learning in mathematics cannot be reached without improved teacher capacity. To accomplish these goals an analysis of current teacher preparation in mathematics is necessary, along with the development of an agenda for improvement. Based on groundwork laid during a meeting in Washington, D.C. in March 2007, the eight members of this study's Mathematics Advisory Group guided the National Council on Teacher Quality's evaluation of the mathematics preparation of elementary teachers. The Mathematics Advisory Group consists of mathematicians and distinguished teachers with a long history of involvement in K-12 education. This statement, followed by the names of the members of the group, summarizes the study's goals:

Our shared perspective on the mathematics preparation needs of the elementary teacher is informed by years of elementary, secondary, and collegiate teaching, preparing preservice and inservice elementary and secondary teachers, developing alternative programs in teacher certification, reviewing mathematics textbooks at all levels, working with professional organizations on mathematics education issues, examining practices in other countries, and advising government agencies and nonprofit organizations on K-12 mathematics standards, education policy, and research.

Our individual experiences with elementary teachers, corroborated by any number of national studies, reveal their limited background in mathematics. There must be a higher standard set for mathematics proficiency in education schools' teacher preparation programs. The minimum first steps toward establishing that standard would be administering assessments that assure general mathematics proficiency as part of the candidate screening process for admission to teacher preparation programs and requiring high standards in coursework containing elementary and some middle school level mathematics topics for program completion.

We have no objections to prospective teachers taking mathematics courses designed for a general college audience, but we strongly recommend teacher candidates take a minimum of three mathematics courses designed specifically for prospective elementary teachers which deal explicitly with elementary and middle school topics. This coursework should be coupled with one mathematics methods course. While we have no objection to an exemption from mathematics coursework for those able to pass a suitable examination, at present there is no such standardized examination that tests the required knowledge.

Our concern that all courses be taught with a high level of integrity leads us to recommend that mathematics courses be taught by instructors with adequate professional preparation in mathematics, and that aligned methods courses be taught in either mathematics or education departments. We consider cooperation and coordination between content and methods instructors to be essential.

Because all possible K-8 mathematics topics are not of equal value for elementary teachers, we have also suggested the amount and distribution of instructional time that is sufficient to cover essential topics. While instructors may supplement a textbook with valuable related materials, a textbook that meets the basic needs of the elementary teacher is necessary in every content course. We recommend that prospective teachers practice their knowledge in classroom situations under close supervision of instructors with high-level qualifications in classroom instruction.

Our nation needs far more elementary teachers who can competently and confidently teach mathematics. Our hope is that better preparation in education schools will strengthen instruction, boost student performance, and increase teacher gratification for a job well done.

The Mathematics Advisory Group

Dr. Richard Askey, *Professor Emeritus of Mathematics*, University of Wisconsin – Madison; *Member*, National Academy

Dr. Andrew Chen, *President*, EduTron Corporation[1], Massachusetts

Dr. Mikhail Goldenberg, *Mathematics Department Chair*, The Ingenuity Project, Baltimore City Public Schools, Maryland

Dr. Roger Howe, *Professor of Mathematics*, Yale University; *Member*, National Academy

Mr. Jason Kamras, *Director of Human Capital Strategy*, Office of the Chancellor, District of Columbia Public Schools; *2005 National Teacher of the Year*

Dr. James Milgram, *Professor Emeritus of Mathematics*, Stanford University

Ms. Robin Ramos, *Mathematics Coach*, Ramona Elementary School, Los Angeles, California

Dr. Yoram Sagher, *Professor Emeritus of Mathematics, Statistics, and Computer Science*, University of Illinois – Chicago; *Professor of Mathematical Sciences*, Florida Atlantic University

Biographical information on members of the Mathematics Advisory Group is found in Appendix A.

1 EduTron is a mathematics and science education products and services company founded by Massachusetts Institute of Technology scientists and engineers.

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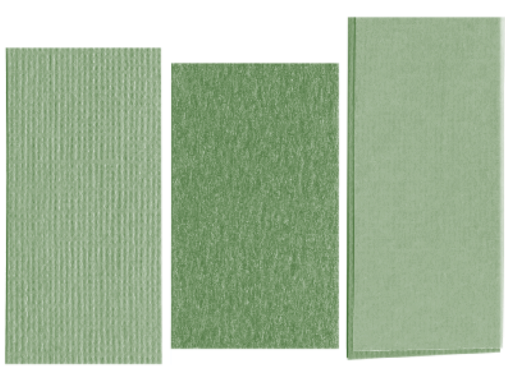


1. INTRODUCTION

WHY NCTQ IS STUDYING TEACHER PREPARATION IN MATHEMATICS

The National Council on Teacher Quality (NCTQ) advocates reforms in a broad range of teacher policies at the federal, state, and local levels in order to increase the number of effective teachers. Because most (approximately 70 percent) of our nation's teachers are prepared in undergraduate education programs, commonly referred to as "education schools," the efficacy of this pathway into teaching remains one of our foremost concerns.

In May 2006, we issued *What Education Schools Aren't Teaching about Reading and What Elementary Teachers Aren't Learning*. After examination of syllabi and texts in undergraduate programs, our study concluded that only 15 percent of the programs in a sample of 72 programs across 35 states were introducing teachers to the explicit, systematic approach to learning how to read that scientists have determined is the most effective way of teaching reading to young children.[2]

In this second study of elementary teacher preparation programs, we examined 257 syllabi and required texts in 77 undergraduate education programs in 49 states and the District of Columbia to ascertain whether the courses in which they are used adequately prepare elementary teachers (kindergarten to grade 5)to teach mathematics.[3]

Because we are committed to lending transparency to and increasing public awareness about the impact of education schools on the quality of our nation's teachers, we plan to continue to examine their programs with a research agenda that has direct and practical implications for policy.

SETTING THE CONTEXT

For several decades some faculty at the institutions that prepare future teachers have been at the center of conflict over how best to deliver K-12 mathematics instruction, a dispute commonly referred to as the "Math Wars." While both mathematicians and mathematics educators on college campuses share in the responsibility of preparing teachers to teach mathematics, they have often been at odds over how to do so. Nevertheless, coordination between the two faculty groups, housed generally in different departments, remains vital to good teacher preparation. Theirs is a tenuous relationship that has not been well served by the heightened sensitivities of the Math Wars.

2 The full report is available at http://www.nctq.org/p/publications/docs/nctq_reading_study_app_20071202065019.pdf

3 "Elementary" education programs found in our sample of 77 institutions match state licensing categories and are predominantly kindergarten (K) through grade 6 (26 programs), but include K-8 (14), grade 1-6 (11), PreK-4 (5), PreK-6 (4), K-5 (4), K-12, self-contained classroom (3), PreK-5 (2), birth-grade 6 (2), grade 1-8 (1), grade 2-6 (1), and K-3 (1).

Compounding this problem is the fact that authority for teacher preparation in most institutions resides within the education school. The administrators of the education school, not mathematicians in the mathematics department, generally determine what mathematics courses will be required and how many. While many academic mathematicians have little direct interest in teacher preparation, those who do often stand frustrated on the sidelines, feeling strongly that prospective teachers need more preparation in mathematics content.

With both sides working hard to find some common ground, the Math Wars appear now to be waning (see "A Chronology of the Math Wars," page 5), and none too soon. The possibility of improved cooperation between both "camps" bodes well for achieving better teacher preparation. Appropriate and adequate content coursework, good pedagogy instruction, and coordination between mathematics content and mathematics methods courses are vital to good teacher preparation, which, for reasons we will now lay out, is urgently needed.

TRACKING THE MATHEMATICAL PROFICIENCY OF ELEMENTARY TEACHERS

Accelerating pre-secondary improvements in mathematics proficiency and setting the stage for secondary improvements could depend on a variety of factors. Because mathematics relies on cumulative knowledge, one of the factors is certainly elementary student facility in the mathematics itself. As a University of Vermont mathematics educator put it, "all of mathematics depends on what kids do in the elementary grades. If you don't do it right, you're doing remedial work all the way up to college."[4] And it is the capability of elementary teachers to instill student proficiency that lays this groundwork.

STUDENT PERFORMANCE

American students at the fourth-grade level are ranked in the middle of the pack of 25 countries on the "world's report card," the Trends in International Mathematics and Science Study (TIMSS), while U.S. eighth graders' performance places them 15th of 45 countries.[5] Steady domestic score increases for fourth graders on the "nation's report card," the National Assessment of Educational Progress (NAEP),[6] have not been matched by TIMSS score increases from 1995 to 2003 (over which period, six other countries have shown improvement).[7] Eighth graders have also shown steady score increases on NAEP,[8] but a sharp increase in the scores of eighth graders on TIMSS in 1999 was not repeated in the most recent TIMSS assessment of 2003.[9]

4 Kenneth I. Gross, Univ. of Vermont, quoted in "Elementary Math Grows Exponentially Tougher: Students, Teachers Tackle Algebra," by Maria Glod, *Washington Post*, 26 December 2007, p. A01.

5 The TIMSS is administered by a U.S. organization; cultural issues are not factors in the test's validity. Scores obtained from Patrick Gonzales, et al., *Highlights from the Trends in International Mathematics and Science Study: TIMSS 2003*, Institute for Education Sciences, National Center for Education Statistics, U.S. Department of Education (2004), Tables 2 and 3, <http://nces.ed.gov/pubs2005/timss03/tables.asp>.

6 Average scores of fourth graders have increased 27 points over the past 17 years. Jihyun Lee, Wendy Grigg, and Gloria Dion, *The Nation's Report Card: Mathematics 2007*, Institute for Education Sciences, National Center for Education Statistics, U.S. Dept. of Education (2007), <http://nces.ed.gov/nationsreportcard/pubs/main2007/2007494.asp>.

7 Gonzales, et al., Table 3.

8 Average scores of eighth graders have increased 19 points over the past 17 years. Lee, et al.

9 Gonzales, et al., Table 4.

A CHRONOLOGY OF THE MATH WARS

Date	Event	Proponents say…	Detractors say…
1989	The National Council of Teachers of Mathematics (NCTM) publishes *Curriculum and Evaluation Standards for School Mathematics* (followed by the *Professional Standards* (1991) and the *Assessment Standards* (1995)). Used in the creation and revision of most state curriculum standards and textbooks.	Emphasis in curriculum placed on understanding and problem solving.	As far as curriculum goes, de-emphasizes practice, standard algorithms, and direct instruction. Gives birth to "fuzzy math." In its focus on curriculum, ignores the entire issue of teacher capacity.
1999	U.S. Department of Education releases *Exemplary and Promising Mathematics Programs.*	The National Science Foundation supported the development of most of the programs identified as exemplary or promising.	As stated in a full page ad in *The Washington Post* listing 200 university mathematicians, the list should be withdrawn and "well-respected" mathematicians should be included in any future evaluations of mathematics curricula.
1999	Liping Ma publishes *Knowing and Teaching Mathematics*, a seminal text on the implications for teaching of a profound understanding of fundamental mathematics.	With a focus on teacher capacity, creates an issue around which a consensus among mathematics educators and mathematicians can grow.	Does not provide an action agenda.
2000	NCTM publishes *Principles and Standards for School Mathematics*. Used in the revision of most state curriculum standards.	Establishes a common set of content topics; defines principles (e.g. equity) and standards. Mathematicians worked as members of each writing group.	Endorses the methods suggested in the 1989 *Standards*. Over-emphasizes the use of technology, and data analysis and probability at the PreK-12 levels.
2001	The National Academies publishes *Adding It Up: Helping Children Learn Mathematics*, exploring how students in Pre-K through 8th grade learn mathematics and recommending how teaching, curricula, and teacher education should change to improve mathematics learning.	Tangible product of the increasing consensus on a wide range of issues in mathematics instruction among mathematics educators and mathematicians.	Widely accepted, but has little impact.
2006	NCTM publishes *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.*	Identifies areas of focus at each level of PreK-8 as critical areas of emphasis. Begins the discussion of important mathematics and creates a coherent curriculum aligned with international curricula.	Identifies a minimalist curriculum. The role of technology is not addressed and the role of processes (e.g. problem solving, connections), data analysis and probability are de-emphasized; over-emphasizes numbers and operations.
2006	President George W. Bush appoints the National Mathematics Advisory Panel (NMP). The NMP's charge: to advise on "the best use of scientifically based research to advance the teaching and learning of mathematics, with a specific focus on preparation for and success in algebra." Report is issued in 2008.	Conceptual understanding, computational fluency, and problem-solving are not competing aspects of mathematics, but mutually supportive, each facilitating the learning of the others. Along these lines, the Panel indicated that instruction should neither be entirely child- nor teacher-centered.	Too narrow in its "critical foundations," very "old school" in its definition of algebra topics, and far too exclusionary in the research accepted for review and use by the Panel.

TIMSS is not the only source of discouraging news. Another international test, the Programme for International Student Assessment (PISA), in 2003 evaluated the mathematics knowledge of 15-year-olds living primarily in Organization for Economic Cooperation and Development (OECD) member countries. The U.S. had average scores below the OECD average in all four areas of mathematics assessed.[10]

Data on how U.S. students fare by the time they finish high school are sketchier, and recent international comparison are not available,[11] but NAEP achievement levels of high school seniors about to enter college or the workplace have been stagnant since the early 1970s, with 2005 test results indicating that only 23 percent were proficient in mathematics.[12]

Whatever gains that have been made in mathematics proficiency in elementary and middle school as reported on the NAEP seem to evaporate in secondary school as students begin to grapple with the conceptual and computational demands of algebra and geometry. Even more important, such mathematics performance gains are in no way sufficient to move the U.S. to the forefront internationally at any level of schooling.

TEACHERS

Many mathematics educators, disheartened by American students' performance in mathematics, now advocate that schools should employ mathematics specialists, alleviating the need to train all elementary teachers in mathematics. Given that so many teachers express such insecurity about their mathematics skills, the suggestion is not without merit. Still, the current staffing arrangement in almost all elementary schools, particularly in the lowest grades, is to employ elementary generalists who must be prepared to teach all core subjects. For the time being and the foreseeable future, elementary generalists are responsible for this foundation. How mathematically proficient then are U.S. elementary teachers, both compared to non-teachers and compared to teachers in other countries?

Many human capital issues in education arise from the lack of competitiveness among teacher preparation programs. Most teacher preparation programs are housed in the less selective colleges and universities. In terms of students who indicate an interest in entering elementary education programs, elementary teachers rank below the average college-bound senior. Averaging those heading for elementary education with the higher-scoring secondary candidates, the 2007 mathematics SAT scores of college-bound seniors who plan to major in education is 483, well below the national average of 515 for all students.[13]

The same relative academic weakness is seen in graduates of education programs. Despite impressive increases in SAT scores since 1994 through 1997, students emerging from elementary education programs

10 National Council of Teachers of Mathematics. *Highlights of the 2003 PISA Math Results*, <http://www.math.hawaii.edu/~tom/hctm/PISA_Highlights.pdf>.

11 The 1995 TIMSS test of students in their last year of secondary school placed U.S. high school seniors fourth from the bottom of 21 countries; among students in their final year of secondary school who are taking or have taken advanced mathematics courses, U.S. seniors were second from the bottom of 16 countries. *Mathematics and Science Achievement in the Final Year of Secondary School*, Third International Mathematics and Science Study, (Boston College: TIMSS International Study Center, 1995), <http://timss.bc.edu/timss1995i/TIMSSPDF/C_Hilite.pdf>.

12 Twelfth-grade scores increased from 304 in 1973 to 307 in 2004. Lee, et al., Fig. 2-4, <http://nces.ed.gov/nationsreportcard/pubs/main2005/2007468.asp>.

13 *2007 College-Bound Seniors Total Group Profile Report*, College Board, (2007), Table 26.
Average SAT mathematics score from College Board, 28 August 2007, <http://www.collegeboard.com/press/releases/185222.html>.

still lag behind the average mathematics SAT scores of all college graduates (542) and have the lowest score (508) for any type of student intending to become a teacher, except for those intending to teach physical or special education.[14] As one mathematics trainer described elementary teachers, "many of them fear math…many of them had trouble with math themselves."[15]

Do education programs then successfully remediate the skill deficiencies of aspiring teachers? The answer is not only unclear to us, but seemingly not known to the programs that prepare teachers, the states that license them, or the districts that hire them. A later section of this study will address the weakness of evidence for mathematics proficiency of graduates provided by the most common end-of-program examinations, essentially the only means that states have to verify the mathematics knowledge of future teachers.

To our knowledge, no large-scale study compares the mathematical proficiency of elementary teachers in the U.S. with teachers in other countries, but recent international comparisons of those countries whose students out-perform our own estimate that foreign students (including prospective teachers) graduate some two or more years ahead of U.S. students in terms of the level of mathematics covered.[16] Highly selective teacher education programs have been cited as a common feature of these countries.[17] Evidence suggests that countries around the world typically have about three times more elementary teachers who major or minor in mathematics or in a related field of science.[18]

Even if elementary education programs were attracting more academically able students, a strong rationale would still exist for greater content mastery in elementary education programs. The fact that many teacher candidates have weak mathematics backgrounds makes a strong mathematics component imperative.

THE CONNECTIONS BETWEEN TEACHER KNOWLEDGE, TEACHER PREPARATION, AND STUDENT PERFORMANCE

While the effect that elementary school teachers have on student achievement gains is considerable — 10 to 15 percent in a single year, particularly in mathematics,[19] and even more when viewed cumulatively[20] — researchers have failed to identify the answer to the question of what mathematics knowledge matters for elementary school teaching. The bulk of

14 Drew Gitomer, *Teacher Quality in a Changing Policy Landscape: Improvements in the Teacher Pool*. (Princeton: Educational Testing Service, 2007), Fig. 21.

15 Vickie Inge, Univ. of Virginia, quoted in "Elementary Math Grows Exponentially Tougher: Students, Teachers Tackle Algebra," by Maria Glod, *Washington Post*, 26 December 2007, p. A01.

16 James Stigler, et al., (1999) and William Schmidt, et al., (2006) as cited in *Knowing Mathematics: What We Can Learn from Teachers*. Research Report Vol. 2, East Lansing: Michigan State University, Promoting Rigorous Outcomes in Mathematics and Science Education, December 2006, p. 3.

17 Michael Barber and Mona Mourshed, "How the World's Best-Performing School Systems Come Out On Top," McKinsey & Co., (2007), <http://www.mckinsey.com/locations/ukireland/publications/pdf/Education_report.pdf>.

18 *Knowing Mathematics: What We Can Learn from Teachers*, p. 13.

19 Barbara Nye, Spyros Konstantopoulos, and Larry Hedges, "How Large are Teacher Effects?" *Educational Evaluation and Policy Analysis*, Vol. 26, N3 (2004), p. 250.

Robert Gordon, Thomas Kane and Douglas Staiger, "Identifying Effective Teachers Using Performance on the Job," White Paper 2006-01, The Hamilton Project, Brookings Institution (2006), p. 8.

20 William Sanders and June Rivers, *Cumulative and Residual Effects of Teachers on Future Student Academic Achievement*. Research Progress Report. (Knoxville: University of Tennessee Value-Added Research and Assessment Center, 1996), p. 6.

Steven Rivkin, Eric Hanushek and John Kain, "Teachers, Schools, and Academic Achievement," *Econometrica* Vol. 73 N2, (2005), p. 449.

research on learning has focused on the knowledge and instructional needs of a secondary mathematics teacher and is nearly irrelevant to settling the question of what an elementary teacher needs to know to be able to teach topics such as place value and fractions correctly and successfully.

From a research perspective, it has been difficult to distinguish the role or prominence that mathematics knowledge plays in the success of a teacher's students. In the 1970s Edward Begle was one of the first researchers to grapple with this topic, stating:

> *There is no doubt that teachers play an important role in the learning of mathematics by their students. However, the specific ways in which teachers' understanding, attitudes, and characteristics affect their students are not widely understood. In fact, there are widespread misconceptions, on the part not only of laypersons but also mathematics educators, about the ways in which teachers influence mathematics learning by their students.*[21]

Begle's own research in this area was followed by more studies (summarized in: "What the Research Says" on page 9) that examined effects of teacher characteristics on student performance. Little is relevant to our study because virtually no research was conducted at the elementary level and proxies were often used to measure subject matter preparation.

Research on teacher preparation is equally equivocal. After an exhaustive review of research, the 2008 National Mathematics Advisory Panel "did not find strong evidence for the impact of any specific form of, or approach to, teacher education on either teachers' knowledge or students' learning."[22]

In sum, the field of teacher education has yet to figure out what mathematics matters most for elementary teaching and to distinguish differences in the effect of specific mathematics content and preparation for teachers.[23]

21 Edward G. Begle, *Critical Variables in Mathematics Education: Findings from a Survey of the Empirical Literature*. (Washington, DC: Mathematical Association of America and National Council of Teachers of Mathematics, 1979), p. 27.

22 United States Dept. of Education. *The Final Report of the National Mathematics Advisory Panel*, (2008).

23 Deborah Ball, Sarah Lubienski, and Denise Mewborn, "Research on Teaching Mathematics: The Unsolved Problem of Teachers' Mathematical Knowledge," in *Handbook of Research on Teaching* 4th ed., V. Richardson (Ed.), (Washington, DC: American Educational Research Association, 2001), p. 453.

WHAT THE RESEARCH SAYS

To date, the most comprehensive studies have adopted production-function models from economics to study home, school, and teacher effects on student achievement gains. As Wilson, Floden, and Ferrini-Mundy (2002) summarize from a review of the existing research:

> The research that does exist is limited, and in some cases, the results are contradictory. The conclusions of the few studies in this area are especially provocative because they undermine the certainty often expressed about the strong link between college study of a subject matter and teacher quality. [24]

Wayne and Youngs (2003) found that mathematics preparation had some effect on student performance (although effect sizes are typically small and interaction effects are substantial), but not at the elementary level — *primarily because almost no research was conducted at the elementary level*:

> In the case of degrees, coursework, and certification, findings have been inconclusive except in mathematics, where high school students clearly learn more from teachers with certification in mathematics, degrees related to mathematics, and coursework related to mathematics. [25]

Several other studies also show the effects of teachers' study of conventional college mathematics, but not when they teach what interests us here, the subjects below ninth grade.[26] These studies are of limited use as the subject matter, grade level, and context vary, few studies replicate what came before them, far too little work is done at the elementary level, and all use proxies to measure subject matter preparation. A single study that comes the closest to linking secondary mathematics preparation and the ability to teach effectively is no more than a course count done by David Monk in 1994. Monk distinguishes between the number of mathematics and mathematics education courses taken by secondary teachers and identifies the point at which the number of additional college mathematics courses ceases to be of value (five) to the high school classroom.

24 Suzanne Wilson, Robert Floden, and Joan Ferrini-Mundy, "Teacher Preparation Research: An Insider's View from the Outside," *Journal of Teacher Education*, Vol.53 N3, (2002), p. 191.

25 Andrew Wayne, and Peter Youngs, "Teacher Characteristics and Student Achievement Gains: A Review." *Review of Educational Research*, Vol.73, (2003), p. 107.

26 Brian Rowan, Richard Correnti, and Robert Miller, "What Large-Scale, Survey Research Tells Us About Teacher Effects On Student Achievement: Insights from the *Prospects* Study of Elementary Schools". Philadelphia: University of Pennsylvania, Consortium for Policy Research in Education Research Report Series, 2002.

Douglas Harris and Tim Sass, "Teacher Training, Teacher Quality and Student Achievement," CALDER Working Paper, (Washington, D.C.: Urban Institute, 2007), p. 32.

Heather Hill, Brian Rowan, and Deborah Ball, "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement," *American Educational Research Journal*, Vol. 42 N2, (2005), p. 396.

2. WHAT ELEMENTARY TEACHERS NEED TO KNOW ABOUT MATHEMATICS AND TEACHING MATHEMATICS

While we heartily endorse high-quality research on how best to prepare elementary teachers, the reform of current preparation programs cannot be postponed until research results are definitive.

With the advice and guidance of its own Mathematics Advisory Group (see Appendix A), NCTQ looked to international benchmarks, national studies, and the views of mathematicians, mathematics educators, cognitive psychologists, social scientists, and economists to help arrive at a coherent prescription for the mathematics preparation of elementary teachers in undergraduate programs.

Numerous expert bodies have made recommendations on preparation:

- According to the 2008 policy recommendations of the **National Mathematics Advisory Panel**, "the mathematics preparation of elementary…teachers must be strengthened…[with] ample opportunities to learn mathematics for teaching. That is, teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to…mathematics, both prior to and beyond the level they are assigned to teach."[27] "A sharp focus [should] be placed on systematically strengthening teacher preparation… with special emphasis on ways to ensure appropriate content knowledge for teaching."[28]
- In July 2005 the **National Council of Teachers of Mathematics** (NCTM) issued a position statement that elementary teachers "should have completed the equivalent of at least three college-level mathematics courses that emphasize the mathematical structures essential to the elementary grades (including numbers and operations, algebra, geometry, data analysis, and probability).[29]
- In 2001 the **Conference Board of the Mathematical Sciences** published a recommendation that prospective elementary teachers take at least nine semester hours on "fundamental ideas of elementary mathematics" in numbers and operations, algebra and functions, geometry and measurement, and data analysis, statistics and probability.[30]

27 *The Final Report of the National Mathematics Advisory Panel*, p. xxi.

28 Ibid., p. 40.

29 National Council of Teachers of Mathematics, "Highly Qualified Teachers: A Position of the National Council of Teachers of Mathematics," (July 2005), <http://www.nctm.org/about/content.aspx?id=6364>.

30 *The Mathematical Education of Teachers*, Conference Board of the Mathematical Sciences: Issues in Mathematics, Vol. 11, (American Mathematical Society in cooperation with the Mathematical Association of America, 2001), p. 8.

- The **Board of Education for Massachusetts**, a reform-minded state in both academic standard-setting and student instruction, issued 2007 guidelines for the mathematics preparation of elementary teachers in which it indicated that it will require strong justification from programs that propose less than nine semester hours for most candidates on basic principles and concepts important in teaching elementary-school mathematics in number and operations, functions and algebra, geometry and measurement, statistics and probability.[31]

These sources provided us with a relatively clear direction to develop the following standards that address the need for teachers to acquire a deep, conceptual understanding of elementary and middle school mathematics topics, as well as essential pedagogical training.

FIVE STANDARDS FOR THE MATHEMATICAL PREPARATION OF ELEMENTARY TEACHERS

Standard 1:

Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding. Required mathematics coursework should be tailored to the unique needs of the elementary teacher both in design and delivery, focusing on four critical areas:

1. numbers and operations,
2. algebra,
3. geometry and measurement, and — to a lesser degree —
4. data analysis and probability.

Standard 2:

Education schools should insist upon higher entry standards for admittance into their programs. As a condition for admission, aspiring elementary teachers should demonstrate that their knowledge of mathematics is at the high school level (geometry and coursework equivalent to second-year algebra). Appropriate tests include standardized achievement tests, college placement tests, and sufficiently rigorous high school exit tests.

Standard 3:

As conditions for completing their teacher preparation and earning a license, elementary teacher candidates should demonstrate a deeper understanding of mathematics content than is expected of children. Unfortunately, no current assessment is up to this task.

Standard 4:

Elementary content courses should be taught in close coordination with an elementary mathematics methods course that emphasizes numbers and operations. This course should provide numerous opportunities for students to practice-teach before elementary students, with emphasis placed on the delivery of mathematics content.

31 Massachusetts Dept. of Education, *Guidelines for the Mathematical Preparation of Elementary Teachers*, (June 2007), p. 4, <http://www.doe.mass.edu/mtel/MathGuidance.pdf>.

Standard 5:

The job of teaching aspiring elementary teachers mathematics content should be within the purview of mathematics departments. Careful attention must be paid to the selection of instructors with adequate professional qualifications in mathematics who appreciate the tremendous responsibility inherent in training the next generation of teachers and who understand the need to connect the mathematics topics to elementary classroom instruction.

Mathematicians and mathematics educators agree on the need for courses that better meet the needs of elementary teachers. With increasing consensus supporting the recommended elementary mathematics curriculum and instruction represented by the 2008 report of the National Mathematics Advisory Panel and NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, there is real potential for a dramatic and productive reconfiguration of teacher education program requirements to meet these five standards.

3. STUDY SAMPLE AND METHODOLOGY

RESEARCH USING SYLLABI AND TEXTBOOK REVIEWS

We base this study of the mathematics preparation of teachers in undergraduate programs primarily on data yielded from course syllabi and all required textbooks, a strategy that we recognize is not without limitations. The strength of a syllabi review coupled with thorough analysis of every required text is that syllabi must have some meaning or they would not be a standard feature of every course. It is reasonable to assume that college instructors give thought and consideration to their syllabi and textbook readings. The combination represents the intended structure of the courses and emphasizes what the instructors view as essential knowledge. That said, we also fully recognize that a course's intended goals and topics as reflected by syllabi, whose limited descriptions are often fleshed-out by textbooks, undoubtedly differ from what actually happens in the classroom. However, typically, *less* than what the syllabi and certainly the texts contain, not more, is apt to be covered in class. Syllabi represent a professor's goal for what he or she ideally wishes to accomplish in a course, but, in reality, inevitable interruptions and distractions almost always leave that goal to some degree unmet.

Obtaining course information from syllabi and textbooks is certainly not the only way to gather data on instruction. We might have supplemented these data with a common methodology used in education research: student interviews or surveys. Interviewing students can be a good source of qualitative data, but it only reveals students' *perception* of a course, and may reflect a lack of awareness of what they will need to be good mathematics instructors.

REVIEW OF LITERATURE

Other than NCTQ's 2006 study on reading, only a few previous studies have used collections of syllabi and texts to examine the quality of preparation provided by education schools. A 2004 study by Steiner and Rozen looking at a group of only 16 education schools was comprehensive in the sense that it included all required courses at each of the institutions studied. They were able to capture a complete picture of what knowledge was considered essential in several key areas (such as reading and education foundations), but they never intended their study to be representative of all subject areas or all education schools.[32] Gettysburg College professor Dan Butin set out to refute Steiner's claims about bias in education foundations courses by collecting 89 syllabi from 85 institutions, comprising what Butin termed a "convenience sample" with results that could not be generalized.[33] Unlike Steiner, Butin did not attempt to collect all of the relevant syllabi from each institution, only what could be readily found on websites.

32 David Steiner and Susan Rozen, "Preparing Tomorrow's Teachers: An Analysis of Syllabi from a Sample of America's Schools of Education," *A Qualified Teacher in Every Classroom? Appraising Old Answers and New Ideas*, Frederick Hess, Andrew Rotherham, and Kate Walsh, Eds. (Boston: Harvard Education Press, 2004).

33 Dan W. Butin, "The Foundations of Preparing Teachers: Are Education Schools Really 'Intellectually Barren' and Ideological?" *Teachers College Record*, (July 2004), <www.tcrecord.org>.

One additional 1995 study by Smargorinsky and Whiting examined the content of English methods courses aspiring teachers took as undergraduates. That effort analyzed data from a sample of 81 institutions. The institutions in this study represented only the third of solicited schools that responded to a request from the researchers to supply their syllabi.[34]

SELECTING THE SAMPLE

Three major criteria guided our sampling strategy of education schools for this study:

1. The sample would include at least 70 institutions that house education schools. A sample of this size represents at least 5 percent of all institutions offering undergraduate elementary teacher certification in the United States and gave us confidence that we would capture the range of practices by a broad cross section of education programs.
2. The sample would include institutions that differed on important characteristics, including size, location, structure, and student body. This allowed us to determine if certain types of education schools are more inclined to require mathematics content courses of various combinations.
3. The institutions would not be asked in advance to participate in the study to prevent selection bias.

Responding to a legitimate criticism of our reading study when programs were not informed that they were included in the study, we contacted each of the institutions repeatedly *after* they were selected, giving them the opportunity to provide additional materials, such as final exams and study guides, in order to provide a more detailed picture of their courses. Few chose to do so.

The final sample of 77 institutions represents programs of all types and in 49 states and the District of Columbia (excluding Alaska), constituting more than 5 percent of those institutions offering undergraduate elementary teacher certification in the United States.

SELECTING COURSES

After selecting the teacher education programs, we then narrowed our focus to the courses we would need to analyze.

We considered three critical factors in this process.

First, based on the conviction of our Mathematics Advisory Group that coursework needs to impart a deep understanding of mathematics topics relevant to elementary teaching, we only analyzed courses that were designed exclusively for prospective teachers and in which elementary content mathematics was addressed. Importantly and in practical terms, there was no need to evaluate mathematics courses intended for the general population of college students, even if aspiring teachers were required to enroll in such courses.

34 Peter Smagorinsky and Melissa Whiting, *How English Teachers Get Taught: Methods of Teaching the Methods Class*, National Council of Teachers of English, (1995), <www.ncte.org>.

Second, if a program offered aspiring teachers the choice of either a Bachelor of Arts (BA) or Bachelor of Sciences (BS) degree, we studied the coursework required to earn the BA degree, the less rigorous path in terms of mathematics preparation. As this study seeks to identify the "floor" for teacher preparation, not the "ceiling," we wanted to identify the course of study that could be pursued by the prospective teacher least interested in or comfortable with mathematics and least motivated to take mathematics courses.[35]

Third, and related to the second factor, if a program required students to select an area of concentration, we studied the coursework required for students who chose to concentrate in a subject *other* than mathematics. Some programs pointed out to us that elementary teachers could choose a more intensive course of study. We deemed the availability of such choices by a program as irrelevant, since in almost all school districts, all elementary teachers, not just those who had chosen to concentrate in mathematics, need to be able to teach mathematics.

OBTAINING COURSE MATERIALS

From January 2007 through March 2007, we gathered the majority of our 257 analyzed syllabi and textbooks according to the following guidelines:

1. We obtained a syllabus for every required elementary content course and mathematics methods course from Internet searches, students enrolled in the courses, or instructors.
2. We excluded syllabi dated earlier than 2006 and obtained a more recent syllabus unless we verified with the institution that the syllabus was still in use.
3. When we obtained multiple syllabi from a single course taught in sections, we randomly selected a single syllabus to represent a course.
4. After collecting the syllabi, we acquired the most recent edition of every text that was required for the course.

We also conducted an e-mail survey of all mathematics content and mathematics methods course instructors to obtain information on class sizes, grading practices, instructor title, exemption mechanisms, practice teaching requirements, and additional course materials, such as exams. Despite repeated solicitations, response was insufficient to draw general conclusions, but we noted information provided and considered it as a supplement to syllabus information.

35 Five programs that require a fifth year for certification were included, three of which are in California, where such programs are standard.

RATING PROGRAMS

Our rubric for rating elementary content courses considers two dimensions: 1) **breadth**, meaning whether programs covered all 12 essential topics as discussed below; and 2) **depth**, meaning whether programs devoted adequate *time* to these topics taken as a whole.

BREADTH: DECIDING WHAT TOPICS ARE ESSENTIAL TO STUDY

We were able to reach a solid consensus as to the essential topics that all aspiring elementary teachers must study based on a comprehensive review of national and international curricula, studies, and policy documents, as well as expert opinion.

The NCTQ Mathematics Advisory Group analyzed several important policy documents that laid out the recommended mathematics topics for elementary content textbooks and coursework, including topics commonly listed in state standards, topics addressed in the National Council of Teachers of Mathematics' (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, and the topics contained in the fourth and eighth grade assessments in the Trends in the International Mathematics and Science Study (TIMSS). (See Appendix C for an explicit comparison of our recommended list of topics and those in the latter two reports.)

The recommended topics that emerged from this review conform to those endorsed in the report of the National Mathematics Advisory Panel as the critical foundations for teaching algebra. The recommended distribution of instructional time also conforms to the Panel's call for focus on the essentials, both in elementary schools and in teacher preparation programs.[36]

Recommended topics are listed on page 17, as well our Advisory Panel's approximations of time in class that needs to be spent on a particular topic, given its complexity and importance:

36 *The Final Report of the National Mathematics Advisory Panel*, p. 20.

The Breadth of Mathematics Content that Elementary Teachers Need

Critical areas	Essential topics	Estimated class time needed
I. Numbers and operations	1. Whole numbers and place value 2. Fractions and integers 3. Decimals (including ratio, proportion, percent) 4. Estimation	Subtotal: 40 hours
II. Algebra	5. Constants, variables, expressions 6. Equations 7. Graphs and functions	Subtotal: 30 hours
III. Geometry and measurement	8. Measurement 9. Basic concepts in plane and solid geometry 10. Polygons and circles 11. Perimeter, area, surface area, volume	Subtotal: 35 hours
IV. Data analysis and probability	12. Probability and data display and analysis	Subtotal: 35 hours
Total Estimated Time Total:		115 hours = roughly three 45-hour courses

HOW WE EVALUATED SYLLABI FOR BREADTH

We collected 126 elementary content course syllabi that were then evaluated by two trained reviewers with mathematical expertise.[37] Each evaluated a syllabus independently for indications that the classroom instruction at least intended to cover all of the 12 essential topics.[38]

When syllabi were too ambiguous to warrant any conclusions about coverage, the reviewers checked textbook pages assigned for class or reading to ascertain the nature of the instruction.[39] When a third reviewer with mathematical expertise ascertained that the pair did not agree on a particular score or a rationale for a score, the pair reached consensus scores and rationales by discussion.

Samples of course syllabi and their scores are contained in Appendix E.

37 A total of 131 syllabi described methods courses.

38 Any supplemental material on assignments and assessments that we had been able to obtain from instructors was also reviewed.

39 If a syllabus did not provide information on course objectives or assignments that would allow the reviewers to make a reasonable judgment, the syllabus was rated unclear. Because the slightest reference to a topic was sufficient to earn points for coverage, few syllabi were rated unclear. Additional credit was not awarded if a topic was covered in two different courses in the same program.

HOW WE EVALUATED TEXTBOOKS FOR BREADTH

There were 18 elementary content textbooks or textbooks used in elementary content courses in our sample. Each of the textbooks was reviewed by a mathematician on our Mathematics Advisory Group to determine the adequacy of its treatment of the 12 essential topics.[40] All but the weakest and/or least commonly used textbooks were reviewed twice in the three critical areas of numbers and operations, algebra, and geometry and measurement.[41] These evaluators assessed the topics in each critical area on the basis of coverage, connection, integrity, the sufficiency and significance of examples, and whether the text addressed methods of teaching.

The Mathematics Advisory Group considers word problems of paramount importance in elementary content coursework. They paid particular attention in their reviews to the sufficiency and appropriateness of word problems.

The rubric for evaluating textbooks, their scores, and descriptions of features of selected textbooks are found in Appendix D.

RATING PROGRAMS ON CONTENT BREADTH WITH SYLLABUS AND TEXTBOOK SCORES

Using the syllabi and textbook scores, we rated each program on how well it covered the 12 essential topics shown above.

The diagram on page 19 provides an example of how an instructional score that was a composite of syllabi and textbooks scores was calculated for a program with three elementary content courses.

As can be seen, no single course passed or failed, although each received a syllabus score based on its coverage of essential topics. The scores in each course were simply added together to obtain a single "course syllabi score."

The textbook score, on the other hand, was an average of the scores of the textbooks used in all courses. Textbook scores for each course were weighted based on the amount of time taken to address each of the four critical areas.[42] For example, the fact that the third course in the diagram on page 19 uses a textbook

40 Instructors frequently require that students obtain activities workbooks and manipulative kits. Reviews of two of the most commonly required workbooks (one by Bassarear and the other by Beckmann) are found in Appendix D in the commentary on their accompanying textbooks.

41 All data analysis/probability portions of textbooks were judged adequate on the initial review and were not reviewed twice. Each of the other three critical areas of a textbook that was used by five or more courses was evaluated twice, even if judged inadequate on the first review. Only if a critical area was judged inadequate and used by fewer than five courses did it receive only one review. Because numerous little-used books (defined as used in fewer than five courses) were judged wholly inadequate in the initial screening, four books were evaluated only once. To the extent possible, second-stage screenings were conducted in critical-area clusters, allowing, for example, one mathematician to review the numbers and operations sections of most of the textbooks.

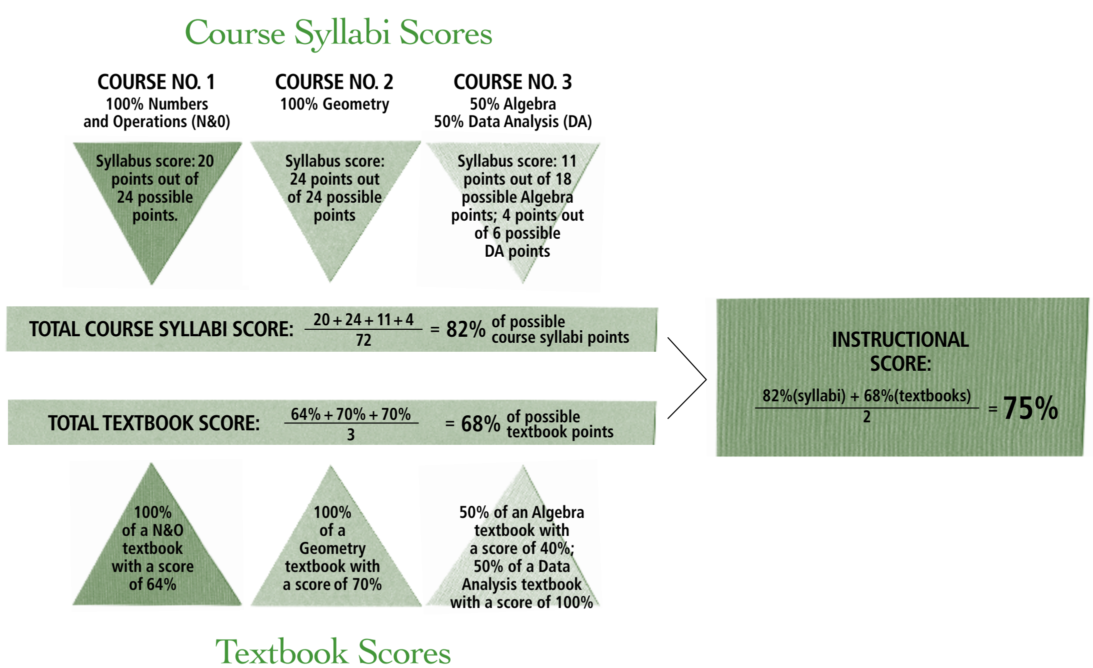
42 Only about one-fifth of syllabi contain detailed course schedules that allow easy calculation of the time allocated to particular topics within critical areas, but about a quarter of courses address only one critical area. We were able to develop rough distribution figures for time spent on each critical area, although we could not ascertain allocation of time among topics within each critical area. These calculated distributions also allowed us to weight textbook evaluations. For example, a review of the numbers and operations section of a textbook is irrelevant for the textbook score of a course in which numbers and operations are not addressed; a review of the geometry section of a textbook should be more heavily weighted than the data analysis section in a textbook score if geometry comprises the vast majority of the instructional time.

that was very adequate for data analysis (with a textbook evaluation score of 100 percent) was irrelevant when considering the textbook score for the portion of the class spent on algebra topics. The algebra textbook evaluation score was only 40 percent. With half of the third course devoted to algebra and half to data analysis, the course's overall textbook score was 70 percent.

Each program's course syllabi score and textbook score were then averaged to produce an instructional score.

While the diagram below depicts scoring of a three-course program, all programs earned a single instructional score reflecting coverage of the essential topics that had been accomplished in one, two, three, or even four courses. This instructional score was used to determine if *programs* passed or failed.

Calculating the Instructional Score for Breadth



DEPTH: DECIDING HOW MUCH TIME TO SPEND ON TOPICS

Our score for the *breadth* of coverage of essential mathematics topics does not reflect the hours of instruction devoted to those topics. For this reason, it is possible for a program devoting one semester to elementary mathematics coursework and another devoting three semesters to have the same instructional score. A high instructional score reflects the fact that essential topics are being addressed during class with the support of a strong textbook. However, this score is insufficient until program **depth** is considered — how much time is being devoted to these essential topics. The process for evaluating depth is described in Appendix B.

Given the limitations of this study, we could not expect to identify the amount of actual class time that instructors spent on each of these 12 essential math topics, but had to consider them as a whole. For example, we could not expect to ascertain if Program A spent adequate time on a single essential topic, such as "equations," but we did estimate the time that would be spent on all three essential algebra topics combined (see page 17).

How much time is appropriate to spend on elementary mathematics content instruction to prepare teachers for the elementary classroom? We looked to our Mathematics Advisory Group, NCTM, and other critical sources (see page 10) to provide the recommended number of hours. As shown in "The Mathematics Content that Elementary Teachers Need" on page 17, they recommended that 115 class hours would be required to cover essential topics. Three one-semester (45-hour) courses provide 135 hours of instruction, accommodating coverage of this 115 hours and leaving 20 hours for other topics. This is the basis for our recommendation that *three* elementary mathematics content courses should be required in most teacher preparation programs.

See Appendix B for a full discussion of rating programs.

HOW WE EVALUATED COURSES IN MATHEMATICS METHODS.

Ideally, our evaluation of mathematics methods course requirements and coursework would have been based on four measures:

1. Did it address *mathematics* methods for a sufficient amount of time?
2. Did it require practice teaching and encourage prospective teachers to focus on mathematics content in their practice teaching?
3. Was it coordinated with mathematics content instruction?
4. Was the required textbook a useful guide to the process of teaching mathematics?

Because the third measure does not lend itself to syllabus analysis, mathematics methods courses were instead evaluated only using the criteria stated below:

- A full-semester (45 classroom hours) mathematics methods course should be required; methods for mathematics and another subject (or subjects) or several levels of schooling should not be taught in the same course unless the semester hours entailed are approximately equivalent to two full-semester courses.
- A mathematics methods course or its associated practicum should provide numerous opportunities for students to experience teaching mathematics to elementary students in diverse settings. Teaching opportunities should be connected to a broad range of evaluative tasks that include the *mathematical content quality* of the lesson and *elementary student work products* as evaluation standards. Evaluative tasks should not simply require "self-reflection" but should involve "presenters," peers, any cooperating teachers, and their instructors.
- The textbook(s) required should include one or more that rose to the top in our ratings of those that address the entire instructional cycle.

Because numbers and operations are the heart of elementary mathematics instruction, sections relevant to numbers and operations in almost all mathematics methods textbooks used by schools in our sample were evaluated by a veteran elementary mathematics coach. The evaluation rubric reflected the perspective of a practitioner rather than an academician on treatment of the tasks of analyzing instructional approaches, planning, teaching, and assessing the efficacy of teaching. The rubric and textbook ratings are contained in Appendix F.

DISCUSSION

The process that we used to rate programs depended on identifying what could be classified as an elementary mathematics content course versus a mathematics course designed for general audiences or a mathematics methods course. Identification proved relatively easy for all but a few courses in our sample, based on consideration of stated course objectives and the type of textbook used. We evaluated a few courses that are hybrids, explicitly having stated that they address both content and methods, although issues unique to them are also addressed in our discussion of recommendations.

Syllabus reviewers did not speculate about the quality of instruction. Indeed, we do not pretend to know whether topics are covered in a manner that might develop "deep understanding" and prepare teachers for the pedagogical challenges of the elementary classroom.[43] However, covering content is an obvious prerequisite to covering content appropriately.

43 Foremost among the additional features of mathematics content classes that our Mathematics Advisory Group consider important are presentations by aspiring teachers of lessons as they might deliver them in an elementary classroom, allowing content experts to provide valuable feedback.

Our overarching rating principle was fairness. We routinely gave the benefit of the doubt, rating syllabi for the merest reference to a mathematics topic, and rating the most recent editions of books even when earlier editions were listed on the syllabus.

Our methods of analysis allows us to evaluate the textbook as an instructional variable, but not the many possible additional materials used by instructors. One might argue that too much emphasis was placed on the quality of the textbook in evaluation, given that instructors could deliver content by a variety of presentations and thereby compensate for any textbook deficiencies. We contend, however, that textbooks should contain the content that meets the basic goals of the course, both during college instruction and as a reference when teaching. Given that several elementary content textbooks earned excellent ratings in review, we see no impediment to use of such texts as a supplement to even the most comprehensive lesson plan.

Finally, we did not evaluate the topics addressed in elementary mathematics methods courses, although our Mathematics Advisory Group suggested that the focus of such coursework should be instructional issues pertaining to numbers and operations, the heart of elementary mathematics. Also, the use of technology in mathematics instruction and learning difficulties experienced by children for whom mathematics is a "third language" are topics that merit consideration in a methods course.

4. FINDINGS

STANDARD 1:

Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding. Required mathematics coursework should be tailored to the unique needs of the elementary teacher both in design and delivery, focusing on four critical areas:

1. numbers and operations,
2. algebra,
3. geometry and measurement, and — to a lesser degree —
4. data analysis and probability.

FINDING 1:

Few education schools cover the mathematics content that elementary teachers need. In fact, the education schools in our sample are remarkable for having achieved little consensus about what teachers need. There is one unfortunate area of agreement: a widespread inattention to algebra.

Unlike most other schools that train professionals, it is clear that the *77* education schools in our sample have not come to any intercollegial agreement on how best to prepare future elementary teachers in mathematics. The variation in requirements across the *77* education schools (which we suspect reflects the variation found across all American education schools) is astounding.

It may be best to illustrate the disparate requirements that we encountered by focusing on four of the schools in our sample. The table below depicts the mathematics-related course requirements at these four schools. The course requirements shown here include both the courses that all students on the campus must take and that satisfy college core curriculum requirements for graduation, as well as the courses that only elementary teacher candidates must take.

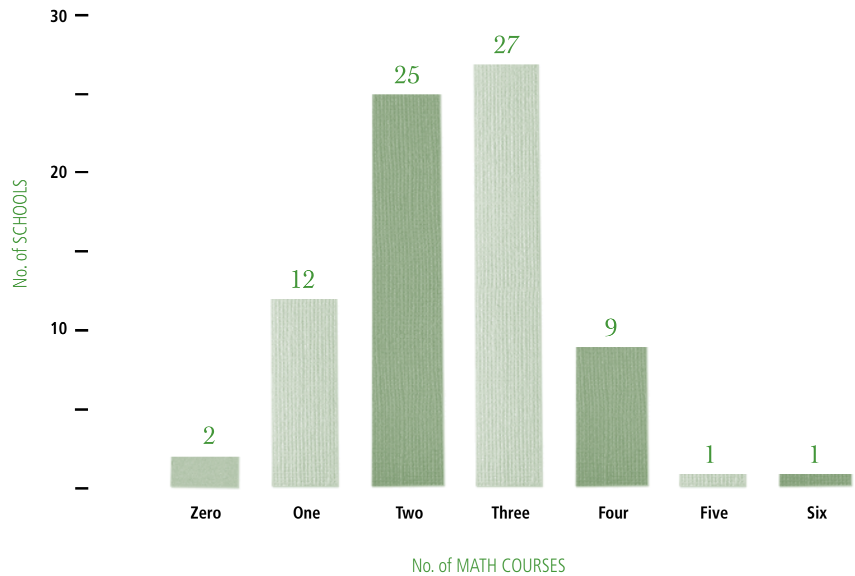
What do colleges require of elementary teachers? It's all over the map

	Metropolitan State College of Denver, CO	Florida International University	St. John's University, NY	Gustavus Adolphus College, MN
1. Math courses designed for any student on the campus, i.e., "general-audience" courses.	**None** required.	A core curriculum requirement of **at least nine semester hours of math of student's choosing** but must include algebra or above, and geometry.	**One course** of student's choosing to satisfy core requirements; **one** course requirement **"by advisement of teacher preparation program"** (6 cr.).	**Two courses**: The Nature of Math (4 cr.) and Elementary Statistics (4 cr.), both satisfy education school requirements and the former satisfies core curriculum requirement.
2. Math courses tailored to the elementary teacher candidate.	**Three courses** required by the education school: Integrated Math I (3 cr.); Integrated Math II (3 cr.); and Math of the Elementary Curriculum (3 cr.).	**One combined content and methods course** required by the education school: Content and Methods of Teaching Elementary Math (3 cr.).	**No course** required by the education school.	**No course** required by the education school.
3. Math methods courses preparing elementary teachers.	**Two multi-subject courses** required by the education school: Integrated Methods of Teaching Science, Health & Math: K-6 (3 cr.) Integrated Science, Health and Math Field Experience K-6 (1 cr.).	**One combined content/methods course** (see above) required by the education school.	**One course** required by the education school: Methods of Teaching Math (3 cr.).	**One course** required by the education school: Elementary Math Methods and Materials (2 cr.).

As an indication of the lack of coherence that we observed, some education schools like St. John's in New York and Gustavus Aldophus College in Minnesota allow aspiring teachers to satisfy most or all of their mathematics requirements through mathematics courses designed for a general audience, failing to differentiate between the mathematical needs of the prospective elementary teacher and those of other students. In other words, these schools do not distinguish the needs of the elementary teacher from the needs of the future accountant or lawyer.

We also observed sizeable variations in the quantity of mathematics coursework that was required. The table below indicates how many semester credits of mathematics coursework each elementary teacher candidate takes to receive an undergraduate degree. Note that these numbers of total credits include both general-audience courses and courses designed for teachers.

Total Number of Math Courses Taken by Prospective Elementary Teachers



Elementary teacher candidates in almost two-thirds of the education schools in our sample are required to take one or more mathematics courses designed for a general audience. Only about a quarter of the schools specify which of these general-audience courses are most likely to be suitable for the elementary teacher candidate. In about 40 percent of the sample programs, students may choose to take any general-audience mathematics course they like, regardless of its relevance to the elementary classroom.[44]

What's wrong with having elementary teachers take courses designed for any college student? Most experts, including the members of our Mathematics Advisory Group and the groups cited on page 10, believe that the practice of educating aspiring elementary teachers through mathematics courses designed for a general audience of college students fails to meet the needs of the elementary teacher. While perhaps counterintuitive, it is, indeed, university mathematicians who have led the charge against these general-audience mathematics courses, arguing that they offer the least *effective and efficient way of training future elementary teachers*. According to Dr. Roger Howe, a mathematician at Yale University:[45]

> *The thesis of Liping Ma and the ongoing work of Deborah Ball have highlighted the specialized mathematical expertise that a teacher must have to teach mathematics effectively. Traditional K-12 instruction, and courses adapted to a general audience from this background, do not address the special needs of future teachers. Future teachers do not need so much to learn more mathematics, as to reshape what they already know. This kind of learning requires a commitment not usually associated with general education courses.*

44 Some programs are entirely non-prescriptive about general-audience mathematics coursework that nonetheless is intended to satisfy education program requirements.

45 Howe has served as a member and chair of the Committee on Education of the American Mathematical Society and was a member of the steering committee for the Conference Board of the Mathematical Sciences' report on *The Mathematical Education of Teachers*. More biographical information is contained in Appendix A.

Many mathematicians argue persuasively that while it is desirable for any college student, including prospective elementary teachers, to take a general-audience course as an elective, education schools are seriously handicapping the capacity of these teachers to be effective by failing to immerse them in those mathematics topics that are directly relevant to the elementary classroom.

For a more detailed discussion of this issue, see the text box "Why general-audience mathematics coursework is the wrong way to prepare elementary teachers for the classroom" and the sample problems "Solving fraction problems versus teaching fraction problems: What's the difference?"

For a discussion of how mathematics courses designed for general audiences and teacher audiences differ, see Appendix G, which describes some possible contrasts using algebraic concepts as examples.

The table on page 26 shows all *77* education schools in our sample, sorting them by required number of elementary mathematics content courses designed for teachers and mathematics methods courses.

Notice that the number of required elementary mathematics courses ranges from zero to four, and the number of required mathematics methods courses ranges from zero to two. With no significant variation in the nature of elementary and middle school mathematics background among students in these programs, and all graduates headed for elementary classrooms, the variation is illogical.

HOW MANY COURSES WILL DO? NOBODY AGREES[46]

(The number of credits in methods courses/The number of credits in elementary math content courses)

Elementary Content Courses (1course = 3 credits)	Methods Courses: No course		One course		Two courses
No course	Hampton U (0/0)	Gustavus Adolphus College (2/0) Southern Adventist U (1.5/0) Southern New Hampshire U (1.5/0) U of Redlands (1.5/0) U of Rhode Island (1/0)	Cal State U, San Marcos (3/0) Cal State U, Stanislaus (3/0) Green Mountain College (3/0) St. John's U (3/0) Saint Joseph's U (3/0) U of Alabama, Birmingham (3/0)	Colorado College (4/0) Greensboro College (4/0) U of Richmond (4/0)[47]	
		Florida International U (1.5/1.5)			
One course		Newman U (2/3) U of New Hampshire (1/3) U of Texas, Dallas (1.5/3)	Columbia College (3/3) Cedar Crest College (3/3) King's College (3/3) MacMurray College (3/3) Seattle Pacific U (2.7/3.3) Utah State U (3/3) West Virginia U, Parkersburg (3/3)		Chaminade U of Honolulu (6/3)
		Albion College (2/4) Valley City State U (2/5) Walla Walla U (1.5/4)		Viterbo U (4/4)	Saint Joseph's College (6/4)
Two courses	Georgia College and State U (0/6)	Arizona State U (1.5/6) Boston College (1.5/6) Calumet College (1.5/6) Concordia U (2/6) Lee U (2/6) Lewis-Clark State College (1.5/6) Norfolk State U (1.5/6) Park U (2/6) Saint Mary's College (1.5/6) SUNY College, Oneonta (1.5/6) The College of New Jersey (2/6) U of Arizona (1.5/6) U of Central Arkansas (1.5/6) U of Memphis (1.5/6) U of Portland (1.5/6) U of South Dakota (2/6)	American U (3/6) Minnesota State U, Moorhead (3/6) U of Louisville (3/6) U of Michigan (3/6) U of Mississippi (3/6) U of Nebraska, Omaha (3/6) U of Nevada, Reno (3/6) U of South Carolina (3/6) U of Texas, El Paso (3/6) West Texas A&M U (3/6)	Iowa State U (4/6) U of Wyoming (5/6)	
	Western Connecticut State U (0/8)	Boston U (1/7)	Benedictine U (3/8)	Metro. State College of Denver (4/7) Radford U (4/7)	
Three courses	Northeastern State U (0/9)	Lourdes College (1.5/9) U of Montana (1.5/9) U of New Mexico (1.5/9)	U of Maryland, College Park (3/9) Wilmington U (3/9)	Indiana U, Bloomington (4/9)	U of Georgia (6/9)
		Western Oregon U (1.5/10)			
Four courses				Towson U (5/12)	U of Louisiana, Monroe (6/12)

46 In designating the number of semester courses (or fractions of semester courses), consideration was given to whether courses focus solely on elementary mathematics content or mathematics methods.

47 Mathematics content in methods courses narrowly missed our standard for awarding credit.

WHY GENERAL-AUDIENCE MATHEMATICS COURSEWORK IS THE WRONG WAY TO PREPARE ELEMENTARY TEACHERS FOR THE CLASSROOM

While the research exploring effects of teacher preparation on student performance generally only looks at secondary level instruction, we can extract from this research the idea that the mathematical knowledge that matters most is what is closer to the content teachers teach and the work that they do.[48] Those findings suggest that the specific nature of the relationship between the mathematics taught to teachers and the mathematics those teachers teach does, indeed, make a difference. Technical command of advanced mathematics may not be particularly helpful for teaching second graders basic ideas about numbers and operations. Simply requiring more mathematics does not necessarily lead to better teaching.

One recent study found significant positive effects on student performance from knowledge that might be gained in mathematics courses that instruct on content relevant to teaching when tested against other general measures.[49] The effect size was comparable to that of the socioeconomic status of the student, which is typically one of the largest predictors of student performance.

Most emphatically, teachers should not repeat the work of elementary school, taking a course, for example, in which they build up their confidence in adding fractions or doing whole number division. Elementary teaching candidates should not merely repeat and reinforce the education they received as youngsters. They also need more than to learn how to teach children to add fractions or do whole number division. Much of the how of teaching is currently within the scope of a mathematics methods course. The mathematics content coursework that elementary teachers need is neither pure mathematics nor pure methods, but somewhere in the middle. It imparts the foundational knowledge of elementary mathematics topics that is helpful to teaching in and of itself, as well as bridging to instruction in the elementary classroom. It is about why you do what you do, what parts are dictated by the mathematics, what by convention, what for efficacy (and why it is efficacious), possible alternative methods, and so on.

Elementary mathematics content courses enable prospective teachers to retrace the steps in their own elementary education, this time acquiring a conceptual understanding that few can effortlessly construct, and differing from the more procedural understanding that is carried away from far too many childhood classrooms.

The need for elementary content courses exists in addition to the need for prospective teachers to be proficient with standard algorithms.[50] As was aptly expressed by the Massachusetts Department of Education in a recent publication, "elementary mathematics is not elementary."[51]

48 See Wilson, Floden, and Ferrini-Mundy (2001), where undergraduate and graduate degree level had no effect (or even a negative effect) and undergraduate major often has a small positive effect, but specific coursework has more consistent and larger effects, and Monk's 1994 study in which effects at the secondary level vary across topics and courses.

49 Hill, Rowan and Ball, p. 399.

50 Research is limited, but evidence that teachers might benefit from a more rigorous and ambitious vision of elementary content (as evident in some international curricula, with multistep problems, non-routine word problems, and high expectations for computational fluency) can be found in several in-service courses for elementary teachers. While randomized and large-scale studies are needed, we note data from the Intensive Immersion Institute of the Massachusetts Mathematics and Science Partnership: 63 to 86 percent of students of teachers who chose to attend immersion mathematics courses with this more rigorous vision of elementary content tied to real pedagogical issues faced in their classrooms were ranked "proficient" on state assessments, increases of between 30 and 49 percent over their peers. A small-scale analysis of the value-added effects of the Vermont Mathematics Initiative (VMI) found that students who were taught by VMI-trained teachers performed better on standard measures of mathematics performance and were less likely to drop out of school than other students. Data on Intensive Immersion Institute provided by Dr. Andrew Chen, EduTron, Winchester, Massachusetts. Data on VMI from *The Vermont Mathematics Initiative: Student Achievement from Grade 4 to Grade 10, 2000 through 2006*, Herman Meyers and Douglas Harris, Paper presented at the Annual Meeting of the American Educational Research Association, New York, NY, March 25, 2008.

51 *Guidelines for the Mathematical Preparation of Elementary Teachers*, pg. 5.

SOLVING FRACTION PROBLEMS VERSUS TEACHING FRACTION PROBLEMS: What is the Difference?

INSTRUCTIONAL ISSUE: Create two models for fraction problems in which students need to carefully consider the relationships of "parts" and "wholes" and how the "whole" for the part of the problem under consideration can change with the circumstances. Use an area model for the first and a set model for the second.

PROBLEM ONE: John had a chocolate bar. He gave 4/5ths to his brother. His brother gave 5/8ths of his piece to his sister. What fraction of John's original candy bar did the sister end up with?

Computation of Answer: 5/8 * 4/5 = 1/2

Foundational Content Knowledge Relevant to Teaching:

- Understanding fraction problems requires understanding the "whole" associated with the fraction, that is, what the fraction is "of." The "whole" in this problem changes. First it is John's candy bar. Then the "whole" is the amount given to his brother. Lastly the question asks the student to return to the original "whole" of John's candy bar.
- The meaning of the "whole" in the area model must be carefully explained as the total area represented by an original figure. In this case when asked to take 4/5th of the "whole," this will be modeled as a sub-region of the original figure with area equal to 4/5th of the area of the figure. When we're required to regard the new region as a new "whole" and take, say 5/8ths of it, the model for 5/8ths will be a sub-region of this new "whole" containing 5/8ths of the area of the new "whole."

Set Model Appropriate for Instruction:

What is the Difference? (CONT.)

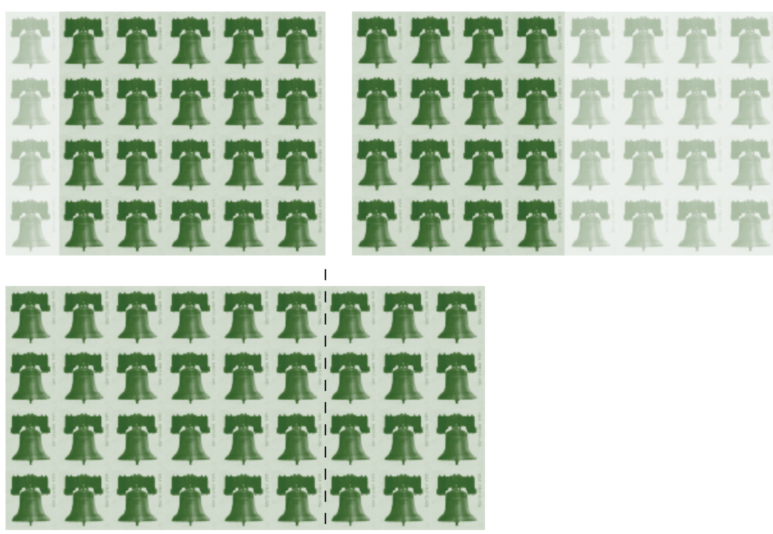
PROBLEM TWO: Ms. Ruiz collected 24 stamps. She glued 1/6th of them into an album. A friend gave her ½ of another collection of 32 stamps. Mrs. Ruiz added them to the pile of stamps that she has not yet glued into her album. What fraction of Mrs. Ruiz's original stamp collection is now not glued into an album?

Computation of Answer: 5/6 (24) + 1/2 (32) = 36; 36/24 = 1 1/2

Foundational Content Knowledge Relevant to Teaching:

- Understanding fraction problems requires understanding the "whole" associated with the fraction, that is, what the fraction is "of." There are two "wholes" in this problem: the original 24 stamps and the friend's 32 stamps. In the second part of the problem, the "whole" to which a comparison of a new quantity in the set is made is the original "whole."
- For the set model the only subsets of a given finite set with n elements that are possible have j elements where $0 \le j \le n$. If we regard a given set with n elements as our "whole," the only fractions we can model are those of the form j/n, and j/n is modeled by any subset of the "whole" that contains j elements. Again, it can happen that the selected subset with j elements becomes a new "whole" at a further stage of a problem. In this case the only fractions of this new "whole" that we can take are those of the form v/j.
- When set models are used in instruction, students may have difficulty understanding the "whole" in such models, particularly in modeling fractions greater than one.

Area Model Appropriate for Instruction:



EVALUATING THE QUALITY OF THE MATHEMATICS PREPARATION OF ELEMENTARY TEACHERS

Few education schools stand out for the quality of their mathematics preparation.

Only 10 schools in our sample (13 percent) rose to the top in our evaluation of the overall quality of preparation in mathematics. These schools require a sufficient number of content courses and use this time to focus on essential and relevant topics.

Education Schools With the Right Stuff

1. University of Georgia – *An exemplary program*
2. Boston College (MA)*
3. Indiana University, Bloomington
4. Lourdes College (OH)*
5. University of Louisiana at Monroe
6. University of Maryland, College Park
7. University of Michigan
8. University of Montana*
9. University of New Mexico*
10. Western Oregon University*

* Although these schools pass for providing the right content, they still fall short on mathematics methods coursework. They do not require a three-credit course dedicated solely to elementary mathematics methods.

Education schools that would pass if more coursework was required

These 25 education schools listed on page 32 appear to focus on essential topics, and tend to do so with strong textbooks. However, they fail to require the necessary amount of coursework that would allow sufficient instructional time to teach these topics thoroughly (nine semester hours, unless the institution has highly selective admission requirements, in which case it would be six semester hours). For example, the University of Nevada, Reno has one of the highest scores of any education school in our sample on the basis of covering all 12 essential topics (see page 17), but requires only six semester credits of content coursework. Programs in this category would only need to boost their requirements by one or two courses of comparable quality to pass. Two institutions — the University of Wyoming and King's College in Pennsylvania — are scheduled to do so.

Education schools that would pass with better focus and textbooks

A relatively small number of schools listed on page 32 require enough coursework but are not utilizing the ample instructional time they have available to address the 12 essential topics (see page 17) and/or they are using substandard textbooks. For example, commendably, Wilmington University is one of the few schools in our sample that dedicates a course to algebra, but our endorsement is qualified because the course dilutes its focus on elementary content algebra by providing, for example, a "high-level overview of calculus," and the textbook supporting instruction is unacceptably weak in algebra.

Five schools need to take an inventory of their elementary content courses and ensure that they are making the best possible use of instructional time.

ARE EDUCATION SCHOOLS PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS?

Education Schools with the Right Stuff

An exemplary teacher preparation program

UNIVERSITY OF GEORGIA

Boston College, MA†
Indiana University, Bloomington
Lourdes College, OH†
University of Louisiana at Monroe
University of Maryland, College Park
University of Michigan
University of Montana†
University of New Mexico†
Western Oregon University†

† Although these schools pass for providing the right content, they still fall short on mathematics methods coursework. They do not require a full course dedicated solely to elementary mathematics methods.

Education Schools that Would Pass if They Required More Coursework

Arizona State University
Boston University
Calumet College of St. Joseph, IN
Cedar Crest College, PA
Chaminade University of Honolulu, HI
Columbia College, MO
Concordia University, OR
Georgia College and State University
King's College, PA
Lewis-Clark State College, ID
Minnesota State University Moorhead
Radford University, VA
Saint Joseph's College of Maine
Saint Mary's College, IN
State University of New York (SUNY) College at Oneonta
University of Central Arkansas
University of Louisville, KY
University of Mississippi
University of Nevada, Reno
University of Portland, OR
University of South Carolina
University of South Dakota
University of Texas at El Paso
University of Wyoming
West Texas A&M University

Education Schools that Would Pass with Better Focus and Textbooks

Benedictine University, IL
Northeastern State University, OK
Towson University, MD
Western Connecticut State University
Wilmington University, DE

Education Schools that Fail on All Measures

Albion College, MI
American University, DC
California State University, San Marcos*
California State University, Stanislaus*
Colorado College*
Florida International University
Green Mountain College, VT**
Greensboro College, NC*
Gustavus Adolphus College, MN*
Hampton University, VA*
Iowa State University
Lee University, TN
MacMurray College, IL
Metropolitan State College of Denver, CO
Newman University, KS
Norfolk University, VA
Park University, MO
Seattle Pacific University, WA
Southern New Hampshire University*
Southern Adventist University, TN*
St. John's University, NY*
Saint Joseph's University, PA*
The College of New Jersey
University of Alabama at Birmingham*
University of Arizona
University of Memphis, TN
University of Nebraska at Omaha
University of New Hampshire, Durham
University of Redlands, CA*
University of Rhode Island*
University of Richmond, VA*
University of Texas at Dallas
Utah State University
Valley City State University, ND
Viterbo University, WI
Walla Walla College, WA
West Virginia University at Parkersburg

* Programs requiring no elementary content coursework at all.

** New coursework requirements are not publicly available.

Education schools that fail on all measures

A near-majority of schools in our sample (48 percent) earned low ratings because they both fail to require enough coursework and fail to even allude to essential topics. Included here are the 15 programs that require no elementary content mathematics courses at all. Simply adding more courses is not a fix, however. Coursework also needs to focus on the 12 essential topics supported by the best available textbooks (see page 36 for textbooks ratings). For example, numbers and operations topics are addressed by West Virginia University at Parkersburg's sole elementary content course, but not comprehensively (fractions, decimals, and integers are ignored). Simply creating two more courses of the same quality and adding them to the existing course will not create viable elementary teacher preparation.

Significant restructuring is needed for 37 programs.

We need to emphasize that we did not make it particularly hard for schools to earn a high rating. Given the acknowledged limitations of what can be learned about a course from syllabi and texts alone, we were inclined to give schools the benefit of the doubt whenever information was ambiguous.

IMPROVING THE FOCUS OF MATHEMATICS PREPARATION FOR ELEMENTARY TEACHERS

How should class time be best spent? Looking at the nature of content to be covered, the NCTQ Mathematics Advisory Group recommended the following allocation of class time to four critical areas:

1. numbers and operations: 35 to 45 percent;
2. algebra: 20 to 30 percent;
3. geometry and measurement: 20-30 percent;
4. data analysis and probability: 5 to 10 percent.

About 15 percent of time remains for other, non-essential topics, or for extended study of the essential topics.

Even with adding a margin of 5 percent to one or both ends of these ranges, *only one program* in our sample addressed these four critical areas in a manner close to this recommended distribution (Saint Joseph's College of Maine). As demonstrated in the chart below (illustrating time allocation among the 61 schools that require at least one elementary content course), the primary reason that schools' time allocations deviate so substantially is that algebra instruction is so anemic. No school devotes more than 25 percent of class time to algebra. Over half of all schools (52 percent) devote less than 15 percent of time to algebra, with another third effectively *ignoring algebra entirely*, devoting less than 5 percent of class time to the area.

Deficiencies in Mathematics Instruction for Teachers

Critical areas	Recommended distribution (hours)	Estimated mean of courses in the sample (hours)	Average hours shortchanged (Estimated for the sample.)
Numbers and operations	40	27	13
Algebra	30	4	24
Geometry and measurement	35	21	14
Data analysis and probability	10	9	1

With the exception of one critical area — data analysis and probability — the average numbers of semester hours that we approximate are devoted to the four critical areas falls well short of recommended amounts.

Even among the 10 schools that rose to the top in our evaluation, instructional deficits still appear to exist, either because coursework focuses too much on geometry or data analysis, or because courses address an excessive number of non-essential topics.[52] As with most teacher preparation, algebra gets short shrift: the average number of hours spent on algebra in these 10 schools is only nine hours, with all programs falling well short of the recommended 30 hours.

FINDING 2:

States contribute to the chaos. While most state education agencies issue guidelines for the mathematics preparation that elementary teachers need, states do not agree on what is needed.

Unlike its more homogeneous counterparts in other countries, teacher education in the United States is molded by a highly fluid combination of state regulations, accreditation requirements, and the policies of individual colleges and universities.

Since all aspects of public K-12 education in the United States are regulated by the states, regulation of the preparation of K-12 teachers, whether at private or public colleges, is also within the purview of states. This contrasts with Hong Kong, Japan, Korea, the Netherlands, and Singapore, all countries whose students out-perform our own in mathematics, where education is nationally controlled and curriculum is approved and accredited by a national agency. Nonetheless, despite the lack of national oversight, states could be more consistent in their regulation. Even at the most superficial level — defining the type of mathematics preparation that elementary teacher should have in terms of coursework, standards, and/or licensure test expectations — there is no consensus in the states.[53]

52 Allocation of time to critical areas was not a factor in the formal evaluation of programs.

53 See NCTQ's *State Teacher Policy Yearbook 2007*, <http://www.nctq.org/stpy/>, for more specific information.

States Guidance is Confusing

18	states have no requirements or no requirements pertaining to specific areas of math:	Alabama, Arizona, Arkansas, Connecticut, Hawaii, Idaho, Iowa, Louisiana, Maine, Maryland, Michigan, Mississippi, Missouri, Nebraska, New Jersey, Virginia, Wisconsin, and Wyoming
1	state has requirements pertaining only to geometry:	Minnesota
3	states have requirements pertaining only to foundations of mathematics[54] and geometry:	Colorado, North Carolina, and Oregon
29	states have requirements pertaining to foundations of mathematics, algebra, and geometry:[55]	Alaska, California, Delaware, District of Columbia, Florida, Georgia, Illinois, Indiana, Kansas, Kentucky, Massachusetts, Montana, Nevada, New Hampshire, New Mexico, New York, North Dakota, Ohio, Oklahoma, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Washington, and West Virginia

Source: NCTQ's *State Teacher Policy Yearbook 2007*, www.nctq.org/stpy

Even within a state, the requirements at individual institutions can bear no obvious relationship to what the state requires. If you compare state requirements with the courses for the four teacher preparation programs listed in the table "What do colleges require of teachers?" on page 22, it is hard to discern in several instances how the education school is satisfying the state's requirements. For example, New York state requires all of its approved teacher preparation programs to prepare elementary teacher candidates in the foundations of mathematics, algebra and geometry. Yet St. John's University, located in New York, does not specifically require any such course or courses for all students.

FINDING 3:

Most education schools use mathematics textbooks that are inadequate.[56] The mathematics textbooks in the sample varied enormously in quality. Unfortunately, two-thirds of the courses use no textbook or a textbook that is inadequate in one or more of four critical areas of mathematics. Again, algebra is shortchanged, with no textbook providing the strongest possible support.

The most commonly used elementary content textbooks are listed below in order of their overall rankings, as rated by our Mathematics Advisory Group.[57] Complete scores and synopses of their evaluations can be found in Appendix D.

54 This can be phrased in many ways, of which these are a few: "understanding of the character and development of number systems and skill in use of numbers" (IN), "understanding and use of the major concepts, procedures, and reasoning processes of mathematics that define number systems and number sense" (MT), "conceptual understanding of the logic and structure of mathematics" (CA).

55 Note that these requirements for algebra and geometry courses may differ from the new Massachusetts requirements for algebra and geometry courses that cover the "basic principles and concepts" in those subjects.

56 The number of published elementary content textbooks is not much larger than our sampling.

57 All textbooks sections on data analysis and probability were determined to be "adequate" in their initial screening.

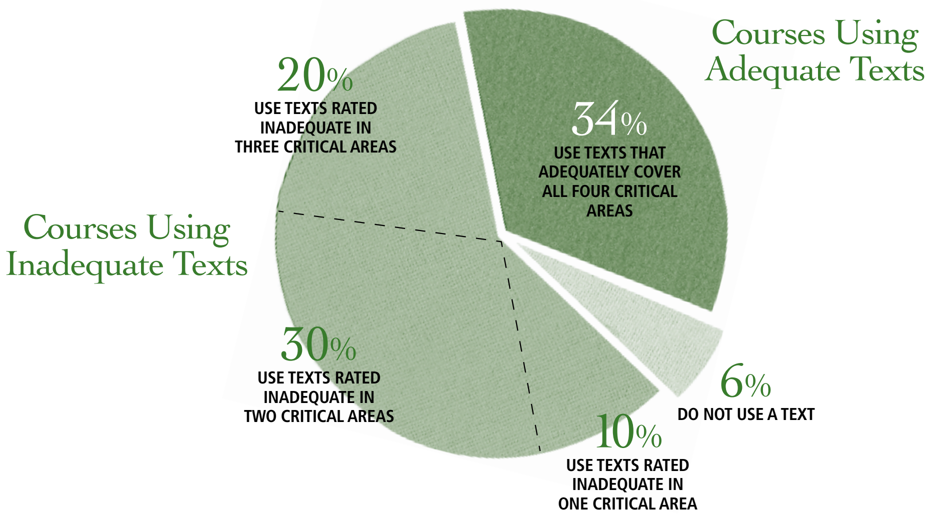
THE TEXTBOOKS TEACHERS NEED: FEW SATISFY

AUTHOR	TITLE	EXPERT RATINGS: Numbers and operations (N&O)	Algebra	Geometry and measurement	Comments	No. of courses in which textbook is used
Sybilla Beckmann	*Mathematics for Elementary Teachers*	+	+	+	Tied for highest score in N&O	11
Rick Billstein, Shlomo Libeskind, Johnny W. Lott	*A Problem Solving Approach to Mathematics for Elementary School Teachers*	+	+	+	Highest score in Algebra	27
Thomas H. Parker and Scott J. Baldridge	*Elementary Mathematics for Teachers* and *Elementary Geometry for Teachers*	+	+	+	Tied for highest score in N&O. Highest score in Geometry. Highest scores on "connecting to the classroom" in ratings of topics in N&O and Geometry. Overall ranking lowered by absence of Data Analysis.	3 (*Elementary Mathematics for Teachers* only)
Gary L. Musser, William F. Burger, Blake E. Peterson	*Mathematics for Elementary Teachers: A Contemporary Approach*	+	−	+		8
Albert B. Bennett, Jr., L. Ted Nelson	*Mathematics for Elementary Teachers: A Conceptual Approach*	+	−	+		8
Phares O'Daffer, Randall Charles, Thomas Cooney, John Dossey, Jane Schielack	*Mathematics for Elementary School Teachers*	+	−	+		7
Calvin T. Long, Duane W. DeTemple	*Mathematical Reasoning for Elementary Teachers*	−	−	+		14
Thomas Sonnabend	*Mathematics for Teachers: An Interactive Approach for Grades K-8*	+	−	+		2
Judith Sowder, Larry Sowder, Susan Nickerson	*Reconceptualizng Mathematics*	−	−	−		3
Tom Bassarear	*Mathematics for Elementary School Teachers*	−	−	+	Lowest scores in N&O and Algebra	22
Charles D. Miller, Vern E. Heeren, John Hornsby	*Mathematical Ideas*	−	−	−	Lowest score in Geometry	6

+ = generally positive − = generally negative

Predictably, the algebra portion of the 11 textbooks shown on page 36 is the weakest, with eight of the 11 textbooks earning scores low enough to label them unacceptable for use in algebra instruction.

Most Courses Use Inadequate Textbooks



DISCUSSION

One might argue that it is possible to teach an excellent course even with one of the less adequate textbooks listed above. However, given that adequate textbooks are available, textbooks used should not have deficiencies that require the instructor to identify assignments of extensive supplementary material to meet the basic goals of the course. Of the 11 textbooks commonly assigned in our sample's programs, the Beckmann text does the best job covering the four critical areas. Absent a better textbook among the handful of elementary content textbooks available that were not reviewed because they are not used by courses in our sample programs, the Beckmann text may seem the best choice for a sequence of content courses. However, Beckmann's algebra section could be stronger. Courses cannot remedy their weaknesses in algebra by supplementing the Beckmann text with the Billstein text, because while Billstein's treatment of algebra is stronger, algebra is not treated discretely but is interwoven throughout the text. The Parker and Baldridge textbooks might also seem to be an excellent choice for the necessary content courses, since they provide the strongest support of any in numbers and operations as well as geometry, but they, too, are weak in algebra and do not address data analysis at all.[58] The fact that no textbook in our sample (and probably no elementary mathematics textbook in general) contains the strongest possible stand-alone algebra section handicaps the preparation of elementary teachers in this vital area.[59]

58 A soon-to-be-released second edition of *Elementary Geometry for Teachers* will include a section on data analysis and probability.

59 Parker and Baldridge plan to publish a third in their series of textbooks that will address only algebra.

STANDARD 2:

Education schools should insist upon higher entry standards for admittance into their programs. As a condition for admission, aspiring elementary teachers should demonstrate that their knowledge is at the high school level (geometry and coursework equivalent to second-year algebra). Appropriate tests include standardized achievement tests, college placement tests, and sufficiently rigorous high school exit tests.

FINDING 4:

Almost anyone can get in. Compared to the admissions standards found in other countries, American education schools set exceedingly low expectations for the mathematics knowledge that aspiring teachers must demonstrate.

While most of the teacher preparation programs in our sample of 77 education schools screen aspiring teacher applicants, they fail to adequately screen on mathematics skills. Programs use a wide range of criteria for this screening: "portfolio artifacts," recommendations, tests of literacy, interviews, ratings of "professional dispositions," and experience with children.[60] The most common criteria cited by these programs, and the only criteria that could possibly bear any relation to mathematics skills, are the following:

- The requirement of a minimum grade point average (GPA) in college coursework taken to date. (Most preparation programs admit students in the spring of their freshmen year.) The majority of programs (65 percent) require GPAs of 2.5 or above.
- A high school or college transcript.
- The requirement of a minimum score on one of many tests including the mathematics portion of the Praxis I, SAT Reasoning Test, ACT, Graduate Record Examination (GRE), or Graduate Management Admission Test (GMAT),or a calculus or statistics Advanced Placement test.

Because neither GPAs nor transcript reviews can accurately reveal an applicant's level of mathematics proficiency, our analysis focuses entirely on whether a program requires objective, accurate tests of mathematical background, as well as how the tests and their results are used.

ENTRANCE TESTS MEASURE SKILLS THAT SHOULD HAVE BEEN ACQUIRED IN MIDDLE SCHOOL.

Fifty-three of the 77 programs (69 percent) require that applicants take a **basic skills test**, typically a three-part assessment of skills in reading, writing, and mathematics. The most common basic skills test is the Praxis I (51 percent) followed by a number of tests that are specific to a state (18 percent), such as the Illinois Basic Skills Test or the Washington Educator Skills Test. However, none of these tests measure *high school level proficiency*, as they address only those mathematics topics taught in elementary and middle school grades.[61]

60 Because information posted on websites about admission and program completion criteria may be incomplete or outdated, we also tried to verify our website findings by surveying the deans or chairs of the education programs in our sample about entrance and exit tests; 39 programs (51 percent) responded.

61 An increasing number — although still a minority — of U.S. students take a first algebra course in middle school. Nonetheless, we classify algebra as a middle school course because it is such in most developed countries.

Most topics on both the Illinois and Washington tests appear to be quite similar to those on the Praxis I, including word problems involving integers, fractions, or decimals, and solving equations, with only a few questions focused on more advanced concepts such as systems of equations. Probably the most challenging questions on Michigan's test involve factoring polynomials and simplifying rational expressions, topics usually introduced in a first-year algebra course.

Consequently, education schools are not testing candidates on their level of high school proficiency. Even substituting the Praxis II for the Praxis I — the former is most often administered at the *end* of the preparation program as a state's condition for licensure — would not address this problem since the Praxis II requires knowledge of elementary and middle school mathematics at a level only slightly deeper than the Praxis I. *A new admissions tool is needed.*

Some programs (16 percent) do not have any assessments required for program admission. They are: Arizona State University, Boston College,[62] Cedar Crest College, The College of New Jersey, Newman University, St. John's University, SUNY College at Oneonta, the Universities of Arizona, Louisville,[63] Montana, and Wyoming, and Valley City State University.

For five institutions (Georgia College and State University, Green Mountain College, Park University, the University of Michigan, and the University of New Mexico), expectations for mathematics proficiency are unclear.

For seven programs using commercial tests (Columbia College, Concordia University, Iowa State University, Lewis-Clark State College, Seattle Pacific University, the University of Portland, Utah State University), we cannot ascertain what the passing score represents.

62 Boston College is "most selective" in its admissions, so most applicants to Boston College's education program have presumably demonstrated basic levels of mathematical proficiency in the general admissions process.

63 Kentucky only requires basic skills tests for admission to less selective institutions.

There are nine programs that require SAT and ACT scores as evidence of mathematics proficiency, or allow their use in lieu of Praxis I or other test sco res.

Is the Standardized Test Score Bar High Enough?

Teacher Preparation Program	Standard
Colorado College	600 on SAT math; 24 on ACT math (required)
Columbia College (Missouri)	SAT or ACT above the national average (option)
Metropolitan State University of Denver (Colorado)	460 on SAT math; 19 on ACT math (option)
Norfolk State University (Virginia)	Unclear, but presumed to be state prescribed score of 1100 SAT (combined reading and math), or 24 on ACT (option)
Southern Adventist University (Tennessee)	22 on ACT (option)
University of Rhode Island	1100 SAT (combined reading and math) (option)
University of Richmond (Virginia)	530 on SAT math; 22 on ACT math (option)
University of Texas at Dallas	550 on SAT math; 22 on ACT math (option)
Western Connecticut State University	450 on SAT math (option)

The table above shows that five programs set a minimum score on the SAT/ACT *option* that is high enough to indicate proficiency in mathematics.[64] Three of the remaining schools set a minimum mathematics score that is below the national average and probably only equivalent to the score needed to pass a basic skills test.[65] Only one program (Colorado College) has a requirement that a student admitted to the education program demonstrate proficiency above the level of basic skills.[66]

STANDARD 3:

As conditions for completing their teacher preparation and earning a license, elementary teacher candidates should demonstrate a deeper understanding of mathematics content than expected of children. Unfortunately, no current assessment is up to this task.

FINDING 5:

Almost anyone can get out. The standards used to determine successful completion of education schools' elementary teacher preparation programs are essentially no different than the low standards used to enter those programs.

64 Above the national SAT math average (515) or the national ACT math average (21); above the national SAT combined reading and math score average (1017) or the national ACT composite score average (24).

65 Western Connecticut State Univ., Southern Adventist Univ. (TN), and the Univ. of Richmond (VA) are in states that allow SAT/ACT exemptions to required basic skills tests. By Missouri law, Columbia College can not exempt a student from a basic skills test requirement based on these scores, but these scores affect the threshold scores on the basic skills test.

66 Colorado College requires that any student not satisfying this requirement take a prerequisite mathematics course to develop proficiency.

Based on both website information and our survey of the education programs in our sample, most programs (87 percent) indicated that they require an exit test in mathematics, with quite a few requiring the test be passed before a candidate can begin student teaching. In almost all cases, these exit tests are the same tests that teachers need to take for state licensure. Only one program reported using an internal exit test that bears no relationship to state licensure. The table below summarizes exit test requirements.

How do programs assess teacher knowledge for program completion?

Exit Tests (n = 77)*

Praxis II (3 types)	State-Specific Licensure Tests	Commercial Tests	Internal Test	No test	Unclear
41 (53%)	21 (27%)	4 (5%)	1 (1%)	3 (4%)	11 (14%)

* There are more than 77 entries because some programs allow a choice among multiple options for exit tests.

There are two major failings of the most commonly used tests. Most states use one of the three elementary level Praxis II tests or a state-specific test. First, these tests either do not report a subscore for the mathematics portion of the test, or second, if they do report a mathematics subscore, it is not a factor in deciding who passes. Under these circumstances it may be possible to answer nearly every mathematics question incorrectly and still pass the test. The only exception to this generalization may be a new licensing test that Massachusetts will unveil in 2009.[67]

The fact that education programs are relying on state licensure tests, such as the Praxis II, as exit tests, and such tests allow prospective teachers to pass without demonstrating proficiency in all subject areas makes it impossible to know how much mathematics elementary teachers know at the conclusion of their teacher preparation.[68]

The content of these exit tests poses another issue. These tests should properly test elementary and middle school content, but *not* at the level one might expect of a competent middle school student. Instead, problems should challenge the examinee's understanding of underlying concepts and apply knowledge in nonroutine, multistep procedures. Though the common assumption may be that the "Elementary Education: Content Knowledge" Praxis II, the most widely required exit/licensure test, evaluates more advanced skills than the Praxis I or evaluates understanding at greater depth, we found little evidence that supports that assumption. Comparisons of the lists of mathematics topics that each addresses and problems found on actual Praxis I and II tests retired by the Educational Testing Service reveal that they cover virtually the same mathematics territory.[69] The fact that these tests are virtually indistinguishable is illustrated by the comparison below

67 The California Subject Examinations for Teachers (CSET), and Oklahoma's General Education Test require passing subscores, but combine mathematics with other subjects in testing. In California's case, mathematics and science are tested together; in Oklahoma, it is mathematics, science, health and fitness, and the fine arts.

68 While not the focus of this study, we note with concern that this same issue is relevant for the Praxis II Early Childhood: Content Knowledge test. There is no mathematics at all in the Praxis II tests for special education.

69 Nine states require another Praxis II — "Elementary Education: Content Area Exercises" — in addition to the content knowledge test or instead of that test. This four-question test contains one question designed to evaluate the capacity to develop mathematics instruction.

of slightly adapted sample test questions taken from Educational Testing Service websites. While these problems may not be illustrative of the complete tests, they indicate that the Praxis II is neither a measure of the content knowledge that should be gained in a teacher preparation program nor an adequate measure of the instructional competence of an elementary teacher. While it may be that the Praxis II requires a demonstration of understanding of slightly more depth than Praxis I, its depth is insufficient.

The education schools in our sample utilize licensure tests from 11 states.[70] Appropriately these tests address elementary and middle school mathematics topics. Only Arizona, California, and Massachusetts post on-line practice tests of sufficient length to conjecture on the level of rigor of the actual tests. Of these, the California Subject Examinations for Teachers (CSET) may be the most rigorous. To the extent that we are able to ascertain it, the test items representing elementary and middle school on these tests generally assess understanding at too superficial a level.

PROBLEMS ADAPTED[71] FROM PRAXIS I AND PRAXIS II

Praxis I Sample Problems

1a. Which of the sales commissions shown below is greatest?

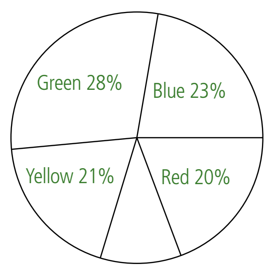
A. 1% of $1000
B. 10% of $200
C. 12.5% of $100
D. 15% of $100
E. 25% of $48

2a. If P/5 = Q, then P/15 =

A. 15Q
B. 3Q
C. Q/3
D. Q/15
E. Q/30

Praxis II Sample Problems

1b. The circle graph below represents the percent of colored gems in a collection. If the collection has a total of 50 gems, how many gems are violet?



A. 2 B. 3 C. 4 D. 5

2b. In the formula x = 20y, if y is positive and the value of y is multiplied by 2, then the value of x is

A. divided by 20
B. multiplied by 20
C. halved
D. doubled

70 Arizona, California, Florida, Georgia, Illinois, Kansas, Oklahoma, Massachusetts, Michigan, New York, and Texas.

71 In each problem one or more numbers were changed. Sources: <http://www.ets.org/Media/Test/PRAXIS/pdf/0730.pdf> <http://www.ets.org/Media/Tests/PRAXIS/taag/0014/mc_questions.htm>.

In recognition of concerns over the nature of its existing licensure test, the Massachusetts State Board of Education has indicated that it will unveil a new MTEL in winter 2009 that is "revised to provide a higher level of assurance that candidates for teacher licenses at the elementary level are sufficiently competent in mathematics" by "strengthening the items to ensure both computational fluency and depth of understanding of the content."[72]

STANDARD 4:

Elementary content courses should be taught in close coordination with an elementary mathematics methods course that emphasizes numbers and operations. This course should provide numerous opportunities for students to practice-teach before elementary students, with emphasis placed on the delivery of mathematics content.

FINDING 6:

The *elementary mathematics* in mathematics methods coursework is too often relegated to the sidelines. In particular, any practice teaching that may occur fails to emphasize the need to capably convey mathematics content to children.

BACKGROUND

Prospective teachers address issues such as analyzing data from student work, planning lessons (developing, differentiating, motivating), and devising ways to assess student learning in general methods courses. Beyond these universal elements of good teaching, mathematics methods coursework focuses on how to provide the child — whose maturity of mathematical thinking develops over time — the capacity for number sense, facility with complicated algorithms, geometric intuition, and an understanding of how to translate quantities and relationships into symbols. None of these can be developed in a general methods class.

Research, albeit limited, indicates the value of mathematics methods courses,[73] and mathematics-specific pedagogy is part of the preparation of mathematics teachers around the world, including in countries such as Singapore, Korea, and Taiwan, whose students out-perform our own.[74]

72 <http://www.doe.mass.edu/mtel/math_sample.pdf>.

73 Begle.

Patrick Ferguson and Sid Womack, "The Impact of Subject Matter and Education Coursework on Teaching Performance," *Journal of Teacher Education*, Vol.44 N1. (1993).

Edith Guyton and Elizabeth Farokhi, "Relationships Among Academic Performance, Basic Skills, Subject Matter Knowledge, and Teaching Skills of Teacher Education Graduates." *Journal of Teacher Education* Vol.38 N5. (1987).

David Monk, "Subject area preparation of secondary mathematics and science teachers and student achievement." *Economics of Education Review*, Vol.13 N2. (1994).

74 *Knowing Mathematics: What We Can Learn from Teachers*, p. 3.

Communications with Mdm Low Khah Gek, Deputy Director, Sciences, Curriculum Planning and Development Division, Ministry of Education, Singapore.

NEARLY HALF OF ALL EDUCATION SCHOOLS DO NOT HAVE A DEDICATED ELEMENTARY MATHEMATICS METHODS COURSE.

Many mathematics educators report that it is difficult to adequately cover all elementary topics in even one methods course, yet many of the programs in our sample (43 percent) do not have one methods course dedicated to elementary mathematics methods.[75] Of these 33 education schools, four do not require a mathematics methods course at all and do not address methods in a significant manner in any coursework; 12 combine mathematics methods with methods in one or more other subjects (most commonly science); 3 combine mathematics methods and content; 13 teach elementary mathematics methods in a course that covers both elementary and middle school mathematics methods;[76] and one combines both different subjects and different levels in one methods course.

Elementary Mathematics Methods on the Sidelines

4	programs with no math methods coursework:	Hampton University, Georgia College and State University, Western Connecticut State University, Northeastern State University[77]
15	programs mixing math methods with methods in other subjects or math content:	Albion College, Boston College, Boston University,* Calumet College of St. Joseph, Florida International University,* Lee University, Lourdes College, Park University, Southern Adventist University, SUNY College at Oneonta, University of Central Arkansas, University of New Hampshire, Durham*, University of Portland, University of Redlands, Western Oregon University
13	programs teaching a combination of elementary and middle school level math methods:	Arizona State University, Lewis-Clark State College, Saint Mary's College, Southern New Hampshire University, The College of New Jersey, University of Arizona, University of Memphis, University of Montana, University of New Mexico, University of Rhode Island, University of South Dakota, University of Texas at Dallas, Walla Walla College
1	program with one course in methods of teaching math, science, and technology in elementary and middle school:	Norfolk State University

* Methods and content mixture in coursework

Ideally, prospective teachers would find that instruction on mathematics content and mathematics methods was well coordinated to enable them to develop their "mathematical knowledge for teaching." The discussion of elementary mathematics content such as fractions and division is made meaningful

75 Five additional programs (Concordia Univ., Gustavus Adolphus College, Newman Univ., Valley City State Univ., and Viterbo Univ.) have mathematics methods courses of only two semester credits.

76 Early childhood (PreK-3) is a very popular related area of certification and four programs combine early childhood, elementary, and middle school mathematics methods in some combination.

77 "Modeling" emphasis in content coursework is a bridge to instruction, but does not constitute methods.

for prospective teachers by its consideration in the context of teaching rather than in the abstract. Likewise, it is difficult to address the pedagogical issues related to instruction concerning these concepts in isolation from a thorough understanding of mathematics.

Coordination of mathematics content and mathematics methods coursework within a program is difficult to assess using syllabi, but we have concerns that it is not common, especially on large campuses. One way to create the opportunity for more coordination is to require concurrent registration for one or more content and methods courses. Only one program in our sample, the University of Wyoming, does so.[78] The University of Georgia requires that students take one of two courses addressing pedagogy in the middle of its three content course sequence.[79]

Few mathematics methods courses fully exploit the instructional potential of practice teaching opportunities. Mathematics methods courses must make transmittal of mathematical content central to the practice teaching experience to fully exploit its instructional potential. Looking at programs that had a course devoted solely to elementary mathematics methods and required practice teaching, we found only six education schools (8 percent) that had syllabi that put content in practice teaching "front and center."

Programs putting *mathematics* at the center of practice teaching

1. Greensboro College
2. University of Georgia
3. University of Louisville
4. University of Michigan
5. University of Nevada, Reno
6. University of Texas at El Paso

In these six programs, we found language such as the following (taken from a rubric for evaluation of practice teaching by the instructor) in syllabi:

> *The unit is based on significant mathematical concepts….*
>
> *The student has used a variety of sources and has demonstrated understanding of what it means to teach for conceptual understanding.*
>
> *The concept map demonstrates that the student is able to identify the key concepts and how they are related.*
>
> *The unit includes formative and summative assessment that shows that it can adequately assess the key concepts identified.*

In contrast, syllabi from other courses requiring practice teaching did not convey the expectation that teaching for mathematical understanding was the fundamental purpose of the exercise. An example is a syllabus describing a practice teaching experience of a minimum of four lessons that are *inquiry-based* and have *driving and sub-driving questions*. Lessons are to be videotaped so that the prospective teacher can use

78 The Integrated Credential Program, an optional certification route at California State University, San Marcos also has this feature.

79 Most programs advise that methods courses be taken after completion of content courses.

them to reflect upon: *what you did, why you did it, what messages you were sending to kids by your words and actions, what you would do differently* and answer many other questions in the same vein, including describing *what part of your teaching philosophy can be seen in your enactment*.

While all of these issues are relevant for reflection and discussion after a teaching experience, consideration of the *mathematics itself* is missing. In no part of this assignment does the instructor convey the expectation that the prospective teacher will consider the mathematical integrity of the lesson and its impact on student performance.

Appendix H contains several sample syllabi illustrating the range of expectations for the practice teaching experience.

STANDARD 5:

The job of teaching aspiring elementary teachers mathematics content should be within the purview of mathematics departments. Careful attention must be paid to the selection of instructors with adequate professional qualifications in mathematics who appreciate the tremendous responsibility inherent in training the next generation of teachers and who understand the need to connect the mathematics topics to elementary classroom instruction.

FINDING 7:

Too often, the person assigned to teach mathematics to elementary teacher candidates is not professionally equipped to do so. Commendably, most elementary content courses are taught within mathematics departments although the issue of just who is best qualified and motivated to impart the content of elementary mathematics to teachers remains a conundrum.

For good or for bad, states are largely silent on the issue of who should teach mathematics content courses or whether courses are to be taught in mathematics or education departments. Only nine states require that all content coursework in teacher preparation programs be taught in the mathematics department. Nonetheless, regulatory action may not be necessary as only eight (6 percent) of the 126 courses that we expected to be housed in the mathematics department were taught instead by the education department.

No matter which department prepares teachers in mathematics, elementary content mathematics courses must be taught with integrity and rigor, and not perceived as the assignment of the instructor who drew the short straw. They should emphasize the concepts that are hard for children to learn and discuss their finer points. The fact that prospective teachers may have weaker foundations in mathematics and are perceived to be more math phobic than average should not lead to a conclusion that the mathematics presented must be watered down.[80]

80 In an effort to "desensitize" teacher candidates who have little confidence in their ability to do mathematics or change their perspective on mathematics, some mathematics educators advocate coursework that addresses novel mathematics topics requiring no advanced mathematical understanding. Two programs in our sample that have such courses utilizing a textbook entitled *The Heart of Mathematics* (Burger and Starbird) may have one or both of these aims. These courses can be valuable electives, but should not substitute for courses addressing essential topics. Alternatively, a text such as *The Heart of Mathematics* can be used as a supplemental textbook in an elementary content course.

FINDING 8:

Almost anyone can do the work. Elementary mathematics courses are neither demanding in their content nor their expectations of students.

We could not evaluate the rigor in mathematics content courses taught in our sample programs using syllabi review because too few syllabi specified student assignments. However, we did make use of assessments collected from mathematics content courses for this purpose. While these assessments may not be representative of all teacher preparation programs in terms of the types of assessments given to prospective elementary teachers, they nonetheless suggest the general level of rigor in all but the less and least selective programs.

A mathematician from our Mathematics Advisory Group reviewed the 23 assessments we had obtained from nine programs in our sample of 77 programs. He classified each question as either appropriate for an elementary or middle school *student* ("elementary classroom level"), or appropriate for a prospective elementary *teacher* ("elementary content level").

The number of test questions that were characterized as elementary classroom level problems on any one assessment in our collection ranged from zero up to 100 percent of all of the questions. The programs in the "most selective" institutions generally managed to avoid testing college students using questions one might find in elementary or middle school classrooms and used "age-appropriate" test questions, but about a third of the questions used by the six programs that are in institutions classified as either "more selective" or "selective" were inappropriate.[81] While there can be a legitimate range of challenge among the problems posed on any assessment, the practice of including more than 10 to 15 percent elementary classroom level problems on an assessment of college students preparing to become teachers is not defensible.

Unfortunately, instructors in programs at the "less" or "least selective" institutions in our sample did not respond to our request for assessment material.

The table on page 48 demonstrates the contrast between the two types of questions, pairing six elementary classroom level problems and six elementary content level problems on a related topic, all taken from *actual* quizzes, tests, and exams used in courses in programs in our sample.

Note that while we judge the elementary classroom level problems as inappropriate for teacher preparation, *citing the elementary content level problems here does not constitute an endorsement*; questions in the "tear-out" test entitled "Exit with Expertise: Do Ed Schools Prepare Elementary Teachers to Pass This Test?" at the end of the report represents the type of elementary content problems that we do endorse.

Additional paired questions of this type are found in Appendix I.

81 About 23 percent of problems in assessments from courses that address numbers and operations or combine numbers and operations and algebra in schools in the "most selective" institutions were "word problems," a highly recommended type of problem to assess true conceptual understanding. This was true of only 5 to 10 percent of problems from such assessments from schools in the "more selective" or "selective" institutions.

CONTRASTING PROBLEMS: The mathematics that teachers need to know – and children do not

Mathematics questions CHILDREN should be able to answer – taken from actual college course assessments.

1a. Which number divides 2711814?
a. 6 b. 8 c. 9 d. 11

2a. Find each. Show the method used.
a. Greatest common divisor (GCD) (60, 132)
b. Least common multiple (LCM) (14, 30)

3a. Write the number 1/13 as a repeating decimal. Show all arithmetic work explicitly.

4a. Which of the following is (2 1/2) ÷ (1/2)?
a. 1 1/4 b. 2 1/4 c. 1 1/2 d. 5

5a. The number 0.0013 is equal to the following:
a. thirteen thousandths
b. thirteen ten-thousandths
c. zero point one three
d. one hundredth and three ten-thousandths

6a. Exactly three-fourths of the students in a certain class are passing. If 24 of them are passing, how many students are in the course?
a. 18 b. 32 c. 36 d. 42

Mathematics questions that are closer to hitting the mark for what TEACHERS should be able to answer – taken from actual college course assessments.

1b. Come up with a test for divisibility by 44 and use it to write down a 20-digit number divisible by 44 whose last digit is 8.[82]

2b. If a and b are positive integers and GCD $(a,b) = 12$, which of the following must be true about LCM (a,b)?
a. LCM $(a,b) = ab/12$ c. LCM $(a,b) > ab/12$
b. LCM $(a,b) < ab/12$ d. none of these must be true

3b. Which one of the fractions below can be written as a terminating decimal?
a. 13/24 b. 51/96 c. 12/52 d. 35/75

4b. Simplify the fraction

$$\frac{(1/2 + 1/3) \div (5/12)}{(1 - 1/2)\,(1 - 1/3)\,(1 - 1/4)}$$

5b. Solve the problem and explain your solution process. Write the number 1.00561616161… as a quotient of two integers (that is, in fractional-rational form). Show step-by-step arithmetic leading to your final, answer, giving a teacher-style solution. Do not simplify your final answer.

6b. The big dog weighs 5 times as much as the little dog. The little dog weighs 2/3 as much as the medium sized dog. The medium dog weighs 9 pounds more than the little dog. How much does the big dog weigh? Solve the problem and explain your solution process.

82 Mathematicians on our advisory group suggested this as a better question: (a) Based on the divisibility rules you learned for 11 and 4, come up with a divisibility test for 44; (b) Write down a 20-digit number that is divisible by 44 — without using "0," "4," and "8" as any of its digits.

DISCUSSION

Are the variations in teacher preparation among the education schools in our sample explained by institutional characteristics?

From the outset, it was our intention to compare institutions with differing profiles to see if certain institutional or program characteristics would make it more likely that mathematics preparation is adequate.

The makeup of our study sample was sufficiently diverse that we could examine the following characteristics:

- Accreditation status of the teacher preparation by the National Council for Accreditation of Teacher Education (NCATE).
- The admissions selectivity of the institution.
- Public versus private institutions.
- Number of teachers the institution graduates each year.
- Percentage of minorities enrolled in the institution.

While few programs passed, all programs had a final instructional score reflecting the combined breadth and depth of mathematics preparation that could be used for statistical analysis.

National accreditation does not add sufficient value. States often require, and about half of the institutions providing teacher preparation have obtained, accreditation by one of two national accrediting organizations, NCATE and the Teacher Education Accreditation Council (TEAC).

The NCATE accreditation process is exhaustive and requires that programs meet standards in all areas, including the mathematics preparation of elementary teachers.[83] However, while accreditation standards and supporting material[84] give guidance as to the nature of which coursework would satisfy the standards, they provide no guidance as to priorities and the necessary extent of coursework:

83 The Association for Childhood Education International (ACEI) accredits institutions in elementary education for the National Council for the Accreditation of Teachers.

84 Candidates are able to teach elementary students to explore, conjecture, and reason logically using various methods of proof; to solve non-routine problems; to communicate about and through mathematics by writing and orally using everyday language and mathematical language, including symbols; to represent mathematical situations and relationships; and to connect ideas within mathematics and between mathematics and other intellectual activity. They help students understand and use measurement systems (including time, money, temperature, two- and three-dimensional objects using nonstandard and standard customary and metric units); explore pre-numeration concepts, whole numbers, fractions, decimals, percents and their relationships; apply the four basic operations (addition, subtraction, multiplication, and division) with symbols and variables to solve problems and to model, explain, and develop computational algorithms; use geometric concepts and relationships to describe and model mathematical ideas and real-world constructs; as well as formulate questions, and collect, organize, represent, analyze, and interpret data by use of tables, graphs, and charts. They also help elementary students identify and apply number sequences and proportional reasoning, predict outcomes and conduct experiments to test predictions in real-world situations; compute fluently; make estimations and check the reasonableness of results; select and use appropriate problem-solving tools, including mental arithmetic, pencil-and-paper computation, a variety of manipulative and visual materials, calculators, computers, electronic information resources, and a variety of other appropriate technologies to support the learning of mathematics. Candidates know and are able to help students understand the history of mathematics and contributions of diverse cultures to that history. They know what mathematical preconceptions, misconceptions, and error patterns to look for in elementary student work as a basis to improve understanding and construct appropriate learning experiences and assessments.

Standard 2.3 Mathematics — Candidates know, understand, and use the major concepts and procedures that define numbers and operations, algebra, geometry, measurement, and data analysis and probability. In doing so they consistently engage in problem solving, reasoning and proof, communication, connections, and representation.[85]

Alternatively, institutions offering teacher preparation may choose to be audited by TEAC, a process in which they provide evidence that they prepare "competent, caring, and qualified" professional educators. This audit process does not include specific coursework requirements, nor does it include any standards.

An analysis comparing the 43 NCATE-accredited programs and the 32 programs that are not NCATE-accredited[86] indicates that while there is a difference in scores on the adequacy of mathematics preparation, it approaches but does not reach statistical significance.[87] The reason for this muted impact may be that NCATE does not address the priority and extent of necessary instruction. Therefore national accreditation is not able to deliver uniformity in coursework requirements. For example, despite their readily evident differences in requirements, all four programs described in the table on page 23 entitled "What do colleges require of teachers?", are accredited by NCATE.

Institutions with less selective admissions are not more likely to provide their less prepared teacher candidates with coursework that is needed.

One might expect to see significant variations between those institutions with high admissions standards and less selective institutions, However, examining mathematics preparation score differences among sample programs grouped into three categories (most/more selective, selective, less/least selective), yielded no significant differences in scores based on selectivity of the institution.[88]

Public institutions are more likely to provide the preparation that is needed than some other types of institutions.

Institutions in our sample can be categorized as public (46), private sectarian (21), and private nonsectarian (10). A statistically significant mathematics preparation score difference is found between public institutions and private nonsectarian institutions.[89]

85 <http://www.acei.org/ACEIElementaryStandardsSupportingExplanation.5.07.pdf>.

86 Two programs have pending applications for NCATE-accreditation.

87 An independent sample t-test of the score differences between accredited ($M = 43.07$; $SD = 27.48$) and non-accredited schools ($M = 32.17$; $SD = 32.16$) had a score of $t(73) = 1.85$, $P = .06$ with an alpha of 0.05.

88 An Analysis of Variance (ANOVA) test was conducted: $F(2,70) = .109$, $p > .05$.

89 An ANOVA for differences among the three categories of institutions revealed a significant difference: $F(2,74) = 3.96$, $p = .05$, with a Scheffe Test determining that the scores of public institutions ($M = 44$: $SD = 26.25$) were significantly higher than those of private nonsectarian institutions ($M = 21.80$; $SD = 24.69$) , $p < .05$.

In terms of all other institutional characteristics, we could not identify factors explaining why some institutions provide the proper preparation and others do not.

Our analyses of the percent minority and the number of teachers graduated at institutions in our sample indicated that these characteristics are not associated with differences in mathematics preparation scores.[90]

90 Grouping programs into two categories, those with a low percentage of minority enrollment (0-25 percent) and those with a high percentage (26-100 percent), revealed no significant difference in scores: low percentage ($M = 39.24$; $SD\ 25.52$), high percentage ($M = 34.32$; $SD = 27.90$), $t(74) = .715$, $p > .05$. An ANOVA compared the 14 top-tier, 17 middle-tier, and 29 lower-tier schools on instructional scores: $F(2,57) = .878$, $p > .05$.

5. RECOMMENDATIONS

We suspect that in several decades we will look back on the current landscape of the mathematics preparation of elementary teachers and have the benefit of hindsight to realize that some programs were poised for significant and salutary change. These are the programs that now have the basic "3/1" framework already in place for adequate preparation, that is, **three** mathematics courses that focus on elementary mathematics content and **one** well-aligned mathematics methods course. Our recommendations here are addressed to professionals responsible for elementary teacher preparation — states, teacher education programs, higher education institutions, and textbook publishers — for remedies to ensure that all programs catch up to the leaders. We also propose initiatives that would build on the 3/1 framework in order to achieve a truly rigorous integration of content and methods instruction.

THE ASSOCIATION FOR MATHEMATICS TEACHER EDUCATORS (AMTE)

The Association of Mathematics Teacher Educators (AMTE) should organize mathematicians and mathematics educators in a professional initiative and charge them with the development of prototype assessments that can be used for course completion, course exemption, program completion, and licensure. These assessments need to evaluate whether the elementary teachers' understanding of concepts such as place value or number theory is deep enough for the mathematical demands they will face in the classroom. They should be clearly differentiated from those assessments one might find in an elementary or middle school classroom.

Current practices for ensuring mathematical proficiency of most elementary teachers have a certain déjà vu quality. Prospective teachers are admitted to education schools having demonstrated basic levels of proficiency in elementary and middle school mathematics; if they are required to take elementary content coursework, it may go little beyond that basic level in its instruction. They are then licensed by passing exams that demand about the same depth of understanding of elementary and middle school mathematics, a depth inadequate for their work.

We offer our "Exit with Expertise: Do Ed Schools Prepare Elementary Teachers to Pass This Test?" at the end of this report as a jumping-off point for the development of a new generation of tests that will drive more rigorous instruction and ensure that teachers entering the elementary classroom are well prepared mathematically.[91]

91 The American Board for Certification of Teacher Excellence (ABCTE) should base the mathematics portion of its elementary education "Passport to Teaching" examinations, an alternative certification route accepted by numerous states, on this new generation of assessments.

STATES

It falls to states to spearhead improvement of education schools by better exercising the oversight authority that they already hold. Most teacher preparation programs will only be able to overcome possible internal resistance or resistance from mathematics departments if state regulations and licensure tests militate for reform.

States must set thresholds for acceptable scores on standardized achievement tests, college placement tests, and high school exit tests. The guiding principle in setting these scores should be to ensure that every aspiring teacher possesses a competent grasp of high school geometry and second-year high school algebra.[92]

While these standards are significantly higher than current ones, they are reasonable. In fact, they still may be lower than what is required of elementary teachers by nations reporting higher levels of student achievement in mathematics. In Hong Kong and Japan, candidates must pass competitive national examinations in multiple subject areas.[93] In Singapore (whose students lead those of all other nations in every international mathematics comparison), the *least* qualified candidates for their graduate education program must still have passed the University of Cambridge "O-level" exam, which assesses high school mathematics topics, in order to then qualify for a program of "remedial" study that includes two mathematics courses.[94]

With the exception of the most selective institutions, there is the quite plausible perception that schools can not raise their admission standards without putting themselves at a disadvantage in the competition for students. The pressure these institutions face to accept a sufficient number of students makes it incumbent upon states to raise the bar for all education schools, not just relegate the task to a few courageous volunteers.

The fact that a large and increasing number of teacher candidates applying for admission to teacher preparation programs are transferring from two-year institutions further underscores the need to establish a uniform and higher threshold for admission.

States need to develop strong course standards and adopt wholly new assessments, not currently available from any testing company, to test for these standards.

To the extent that states regulate the mathematics preparation of elementary teachers, they do so with either coursework requirements, standards (which often take on meaning only in reference to the capacity to teach to the elementary curriculum itself), or some combination of both approaches. As we demonstrated in our earlier discussion of state standards, only the combination of standards *and* coursework requirements ensures that education schools do not decide independently, and all too often inappropriately, what should

92 A recent study (D. Harris and T. Sass, "Teacher Training, Teacher Quality and Student Achievement," CALDER Working Paper, Washington, D.C.: Urban Institute, 2007) indicating that SAT math scores did not have a significant effect on performance of elementary students controlled for all undergraduate coursework, arguably nullifying the score's effect.

93 Wang, et al., p. 18.

94 The more qualified half (who take only a one-year preparation program) pass the A-level, covering more advanced topics, including calculus. For a description of the University of Cambridge examinations, see <http://www.cie.org.uk/qualifications/academic/middlesec/>.

be taught. But even this combination, absent a test, provides no assurance that education schools are teaching to the necessary standards. **A unique stand-alone test of elementary mathematics is the only practical way to ensure that the state's expectations are met**.

Only one state, Massachusetts, is on the road to creating a regulatory framework that accomplishes these goals, goals that should be shared by the entire nation. The Massachusetts General Curriculum test that teachers must pass includes elementary mathematics topics such as numbers and operations, functions and algebra, geometry and measurement, and statistics and probability. Importantly, Massachusetts is not going to allow anyone to use high scores on other parts of this test to get around the need to perform well on the mathematics section. A new, separately scorable mathematics section will be unveiled in winter 2009:[95] Regulations state:

> *Candidates shall demonstrate that they possess both fundamental computation skills and comprehensive, in-depth understanding of K-8 mathematics. They must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense.*

The state has also issued *Guidelines for the Mathematical Preparation of Elementary Teachers*, which specifies that teacher programs should require at least three, ideally four, mathematics courses for elementary and special education license candidates.[96]

All other states should follow suit.

States need to eliminate their PreK-8 certifications. These certifications encourage education schools to attempt to broadly prepare teachers, in the process requiring too few courses specific to teaching any grade span.

While PreK-8 preparation is theoretically possible, and may even be desirable, institutions devote fewer courses than would be needed to provide sufficient preparation for all of these grades. Currently, 23 states offer some form of PreK-8 certification.

EDUCATION SCHOOLS

Education schools should require the coursework with which aspiring elementary teachers can begin to develop a firm but flexible understanding of elementary and middle school mathematics topics and the capacity to instruct on elementary topics. For most we recommend a 3/1 framework: **three** mathematics courses addressing elementary and middle school topics and **one** mathematics methods course focused on elementary topics and numbers and operations in particular.

Education schools should make it possible for an aspiring teacher to test out of mathematics content course requirements using a new generation of standardized tests that would evaluate mathematical understanding at the requisite depth.

95 <http://www.doe.mass.edu/lawsregs/603cmr7.html?section=06>.

96 <http://www.doe.mass.edu/news/news.asp?id=3801>.

The higher education institutions in our sample require an average of 2.5 courses in mathematics, only slightly below our recommendation of three elementary content mathematics courses, although much of that coursework bears little relation to the mathematics that elementary teachers need. Institutions, provided they are willing to redirect their general education requirements to more relevant coursework for the elementary teacher, can quicly move towards meeting this standard by substituting elementary content mathematics courses for current requirements.

As the mathematical foundations of prospective teachers improve with higher entrance standards, less elementary content coursework may be required. Further, we acknowledge that institutions that report highly selective admissions may be able to meet desired instructional standards with a mathematics methods course accompanying only two content courses.

Algebra must be given higher priority in elementary content instruction.

As the National Mathematics Advisory Panel made clear in its 2008 report, while proficiency with whole numbers, fractions, and particular aspects of geometry and measurement are the "critical foundations of algebra," adequate preparation of students for algebra requires that their teachers have a strong mathematics background in those critical foundations *as well as* algebra topics typically covered in an introductory algebra course.[97]

While elementary teachers do not deal explicitly with algebra in their instruction, they need to understand algebra as the generalization of the arithmetic they address while studying numbers and operations, as well as algebra's connection to many of the patterns, properties, relationships, rules, and models that will occupy their elementary students. They should learn that a large variety of word problems can be solved with either arithmetic or algebra and should understand the relationship between the two approaches.

With student readiness for algebra important enough to be specifically mentioned in the presidential charge to the National Mathematics Advisory Panel, the lack of attention paid to this subject in courses for prospective elementary teachers must be remedied.

Education schools should eliminate the following: mathematics programs designed for too many grades, such as PreK-8, the practice of teaching methods for science or other subjects as companion topics in mathematics methods coursework, or the practice of combining content and methods instruction if only one or two combined courses are required.

Teacher preparation programs do a disservice to the mathematics that future elementary teachers need by trying to accomplish too many instructional goals at the same time.

Education schools should reconfigure course sequences intended to prepare teachers for broadly defined certification areas from early childhood through middle school. Even with ancillary course requirements or areas of concentration, these programs will likely fail to adequately address the requirements for teachers

97 *The Final Report of the National Mathematics Advisory Panel*, p. xviii.

at each level, including elementary. For example, PreK-8 programs often have a single three-credit course covering mathematics methods for both elementary and middle school levels. While combining the two may be a good idea, six semester hours would be required to do justice to the material.

The common practice of combining methods relating to instruction in several different subjects in one course is inadvisable. Elementary science — the subject most commonly paired with mathematics in a methods course — is not mathematics-based at the elementary level and does not create any instructional efficiencies in a mixed course.

While we endorse integration of content and methods instruction (as was done by four education schools in our sample), integration is no substitute for adequacy. Teacher preparation programs attempting to integrate instruction should require a combined total of 12 semester hours in content and methods preparation.

Five-year teacher preparation programs, such as those found in California, need to be restructured if they are going to meet the content needs of elementary teachers.

Four of the teacher preparation programs in our sample are five-year programs (Hampton University, California State University, San Marcos, California State University, Stanislaus, and the University of New Hampshire-Durham).[98] The five-year model for teacher preparation, whereby prospective teachers complete coursework for an undergraduate major taking the same courses as would any other major in that subject and then devote a fifth year to courses about teaching and learning, does not accommodate coursework in elementary mathematics topics. To be adequately prepared, a prospective teacher would have to complete the teaching-specific content coursework needed as an undergraduate, which flies in the face of the notion that the undergraduate preparation in this model is separate from the teacher training. For that reason, these programs as currently structured are inadvisable for the appropriate preparation of elementary teachers for teaching mathematics.

HIGHER EDUCATION INSTITUTIONS AND TEACHER EDUCATION PROGRAMS

On too many campuses, teacher education is regarded by university professors and administrators as a program that is beneath them and best ignored. The connection of our national security to the quality of the teachers educating new generations of Americans goes unrecognized. Were teacher education programs to receive more university scrutiny, and demands made that they be more systematic — neither of which is an expensive proposition — change could be dramatic.

Higher education institutions housing education schools must take the lead in orchestrating the communication, coordination, and innovation that would make the preparation of elementary teachers for mathematics instruction coherent. With many variations possible depending on the institution, our recommended coursework might entail coordination of instruction offered by as many as four different educators in two departments. While we urge every instructor and department to make changes in coursework, the value of such changes will be much enhanced when they are part of an institutional reform initiative.

98 Most students at the University of Redlands take five years to complete the program, but it can be done in four.

Much of what has to be changed about the preparation of teachers connects to decisions regarding instruction in mathematics courses (e.g., textbook selection, the priority attached to algebra, establishing more rigorous standards and making practice lesson presentations a central feature of instruction) and mathematics methods course (e.g., coordination with content courses — possibly through concurrent registration — emphasizing the *mathematics* in mathematics methods, especially in practice teaching). Many changes cannot be made in isolation and most will not be undertaken without explicit encouragement by institutional leadership.

The collaboration among different departments required by institutions that have established U-Teach secondary level teacher preparation programs has not only resulted in spin-off collaboration in elementary education programs, but provides a model for the arrangements that are needed in all institutions preparing teacher at any level.[99] At Louisiana State University, for example, the highly successful Geaux Teach program for secondary teacher preparation — a four-course sequence team-taught by content and methods instructors as well as mentor teachers — has a parallel program at the elementary level.[100] In the elementary teacher preparation program, students take 12 credits of elementary content mathematics courses covering all four critical areas of mathematics, with mentored field work included in the coursework beginning in their sophomore year.[101] The final course in this mathematics sequence is taken in conjunction with a six-credit elementary mathematics methods course, allowing content and methods instructors to teach collaboratively.

By itself, leadership from the education department is not sufficient for improving instruction in the content courses elementary teachers need in mathematics. We learned that such courses are frequently relegated to junior or adjunct faculty, or graduate students who may not appreciate the importance of these courses and who consequently do not do justice to the material. Mathematics departments must find the means to staff elementary content courses with instructors with adequate professional preparation in mathematics and ensure that instruction is rigorous and relevant.

TEXTBOOK PUBLISHERS

Several elementary content textbooks (particularly those by Parker and Baldridge, and Beckmann) are excellent and we recommend their use, but content textbooks that are more consistently good across all topics are still needed. We look forward to the completion of the anticipated Parker and Baldridge three-textbook series that will cover all the areas of mathematics that are critical for teacher preparation. In addition, professionals dedicated to improvements in elementary teacher preparation should collaborate to develop a textbook that can serve as a resource both in content and methods coursework. This ideal "combo-text" would augment a core of solid mathematics content with discussion of a process for continuous improvement of instruction focused on student learning.

99 <http://uteach.utexas.edu/>.
100 <http://www.lsu.edu.secondaryed/>.
101 A three-credit college algebra course is also required.

We attempted to triangulate on the missing textbook bridge between contents and methods by having a methods textbook reviewed for its content coverage and several content textbooks reviewed for their methods perspectives. Neither exercise produced a recommended text that met both needs.

A mathematician reviewed the methods textbook, *Elementary and Middle School Mathematics: Teaching Developmentally* (John Van de Walle). This text is used in about one-third of the mathematics methods courses in the teacher preparation programs in our sample, possibly because it discusses mathematical content in the context of the development of children's mathematical thinking.[102] Using the same rubric for evaluating the *content* textbooks in our study, he rated it "inadequate." (See Appendix D for the rubric and a summary of the evaluation).

We asked a veteran elementary mathematics coach and trainer to evaluate the three most highly rated content textbooks *Mathematics for Elementary Teachers* (Beckmann), *Elementary Mathematics for Teachers* (Parker), *A Problem-Solving Approach to Mathematics for Elementary School Teachers* (Billstein) (the most commonly used textbook in elementary content courses in programs in our sample), and a new textbook, *Reconceptualizing Mathematics* (Sowder). Using the same rubric the reviewer had used to evaluate *methods* textbooks (see Appendix F), none of the texts was noted for addressing stages of children's mathematical thinking and for providing teachers with a process for sustaining professional growth focused on student learning. As the reviewer put it, "where are the children?" The children were "found" in *Reconceptualizing Mathematics*, which aims to develop the deep understanding of mathematics needed by elementary teachers; unfortunately, none of the three mathematicians that reviewed this text "found the math."

Our question and challenge to the textbook publishing community: Shouldn't the mathematics content *and* "the children" — discussion and practice of methods for teacher growth in the face of children's learning — be integrated in at least one textbook in a manner satisfactory to both mathematicians and mathematics educators? The "Addendum on Classroom Practice" found at the conclusion of only the first chapter of Parker's *Elementary Mathematics for Teachers* may provide a model for how content and methods might be combined in this textbook.

102 It is also used in Singapore's elementary teacher preparation program.

6. CONCLUSION

American elementary teachers as a group are caring people who want to do what is best for children. Unfortunately, their mathematics instruction leaves far too many of them ill-equipped to do so. We are confident that the education schools that rose to the top in our evaluation process are preparing teachers *relatively* well compared to the majority of education schools in this study, which rated so poorly. Their teachers stand readier than most to forestall the frustrations of youngsters leaving the familiar world of the counting numbers and dealing with the debut of division with fractions. Nonetheless, the standards against which these education schools were judged only lay a solid foundation. Further improvement is still necessary. A deeper understanding of elementary mathematics, with more attention given to the foundations of algebra, must be the new "common denominator" of our preparation programs for elementary teachers. We are only at the beginning of the process of seeing how that new measure might be calculated.

Most of the many mathematicians and mathematics educators we consulted for this study share a vision of teacher preparation programs that are no longer the refuge of math-averse students and in which remediation does not overwhelm efforts to immerse students in a compelling blend of mathematic content and pedagogy. The realization of this vision will require considerably more in the way of building blocks and design plans than we were able to accommodate in this study.

All reform efforts hinge on ensuring that applicants for teacher preparation programs have a firm grounding in mathematics. Admitting candidates who are unwilling or unable to successfully complete a standard high school mathematics program means admitting people whose own elementary and middle school education has failed them, an *a priori* disqualification for teaching at those levels.

Turning to the architecture of reform in education schools, much remains unsettled: the structure and departmental home of courses in which the appropriate instruction can be delivered, the means of integrating content and methods instruction and the professional training for those who can best convey the amalgam of the two, and the nature of textbooks with which such integrated instruction might be supported. We applaud innovative institutions that seek to address those unsettled issues, such as Louisiana State University, with its combined content and methods instruction in its elementary education program.

As we move forward with reforms, we hope to see high-quality research providing evidence of the effects of all teacher preparation programs, innovative and otherwise, on the performance of both their graduates and their graduates' classrooms.[103] (Indeed, the use of classrooms for the mathematics preparation of prospective teachers as the seedbeds for scholarly research may be one of the strongest forces driving their

103 One such study has examined the effects of Louisiana mandates that those enrolled in teacher preparation programs take more content-specific Praxis tests, and that programs work toward accreditation and align their programs with state and national PreK-12 content standards and standards for teachers." Vaishali Honawar, "Gains Seen in Retooled Teacher Ed," *Education Week* Vol.27 N10, (31 October 2007), pg. 1.

improvement.) Because conveying the difference between a superficial and a deep understanding of the mathematics all of us learned as youngsters is so difficult in the abstract, we offer our rudimentary "Exit with Expertise: Do Ed Schools Prepare Elementary Teachers to Pass This Test?" as a tool to help policymakers and all others understand what mathematics preparation must be designed to achieve. We welcome its improvement by the community of professionals who prepare our elementary classroom teachers.

Until such time as an improved instructional model is developed, education schools should increase the efficacy of existing content courses by: intensifying instruction on essential topics with the "laserlike focus" endorsed by the National Mathematics Advisory Panel for K-12 mathematics instruction, selecting the best of current textbooks, and setting high standards for student performance in courses and in exit tests. The prospect that mathematics specialists will become increasingly common in elementary classrooms due to initiatives promoted by groups including the National Academies does not change this imperative for improvement since those specialists can emerge from the same courses and programs as regular elementary classroom teachers.[104] The reforms that will make such teachers more mathematically competent could improve mathematics specialists as well.

While it is encouraging that six education schools in our sample informed us that their requirements for mathematics courses would be increasing in the next few years[105] (and only one mentioned a decrease[106]), this rate of change is simply too slow. With mathematicians and mathematics educators sharing a new consensus about K-12 mathematics instruction, the pace of improvement in the substance and process of teacher preparation can accelerate. An ever increasing number of elementary teachers must walk into their classrooms with the self-assurance that comes from a firm understanding of elementary mathematics, even those who as children left classrooms with their confidence shaken.

Teacher preparation programs are properly responsible for equipping elementary teachers to navigate the mathematical demands of the classroom. Yet as hopeful as we are that the pace of dramatic reforms in teacher preparation will be rapid, many mathematically weak graduates of preparation programs will join their counterparts among the ranks of current teachers. Sustained inservice training directed by mathematicians and mathematics educators is essential to imbue the practice of those professionals with a deeper conceptual understanding. Numerous training programs for current teachers such as the Intensive Immersion Institute of the Massachusetts Mathematics and Science Partnership, the Vermont Mathematics Initiative, and the training associated with a new software-based curriculum entitled "Reasoning Mind"show promise for dramatically increasing the mathematical competence of their graduates. They should be expanded and replicated.[107]

104 Committee on Prospering in the Global Economy of the 21st Century: An Agenda for American Science and Technology, National Academy of Sciences, National Academy of Engineering, and Institute of Medicine, *Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future*. (Washington, D.C.: National Academic Press, 2007).

105 Green Mountain College (VT), King's College (PA), Saint Joseph's College (PA), Univ. of Wyoming, Boston College (MA) and Boston University (MA). The latter two programs provided us with materials on their new coursework and our evaluation reflects their enhanced programs. Green Mountain College (VT) did not respond to our requests for information on new requirements.

106 MacMurray College (IL)

107 The Intensive Immersion Institute trains about 200 teachers per year, most teaching grades 4-8 in a 65 hour course, <http://www.doe.mass.edu/omste/msp/fy04projectsum.html>; the Vermont Mathematics Initiative trains about 75 teachers per year, all teaching K-8, in an 80 hour or a more extended course, <http://www.uvm.edu/~vmi/>; Reasoning Mind trains 125 teachers per year, most teaching grade 5, in a course totaling 120 hours, <http://www.reasoningmind.org/>.

As several prominent mathematics educators have noted, we are now on a treadmill in education. We fail to teach mathematics well, and our weak students become the next generation of adults, some of whom become the teachers who produce the next crop of weak students.[108] With a new and higher "common denominator" for the mathematics preparation of elementary teachers in undergraduate education programs, we can finally jump off this treadmill.

108 Deborah Ball, Heather Hill and Hyman Bass, "Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough to Teach Third Grade, and How Can We Decide?" *American Educator*, (Fall 2005), p. 14.

APPENDICES

APPENDIX A: BIOGRAPHIES OF NCTQ MATHEMATICS ADVISORY GROUP

RICHARD ASKEY, PhD

Richard Askey is an emeritus professor at the University of Wisconsin, where he has taught since 1963. He is a Fellow of the American Academy of Arts and Sciences and an Honorary Fellow of the Indian Academy of Sciences. He was elected to the National Academy of Sciences in 1999.

Professor Askey's research has primarily been in special functions, which are extensions of the functions studied in high school. In addition to many research papers, he coauthored what is now one of the standard books on special functions. More recently he has become involved in issues regarding mathematics education, and was on a plenary panel at the 10th International Congress on Mathematics Education.

He has reviewed many mathematics education reports both nationally and for various states. He was an Edyth May Sliffe Award winner for his work with high school students.

Dr. Askey received his undergraduate degree from Washington University, his master's degree from Harvard University, and his PhD from Princeton University.

ANDREW CHEN, PhD

Dr. Andrew Chen is the President of EduTron Corporation. Before founding EduTron he was a physics professor and a principal research scientist at the Massachusetts Institute of Technology. He currently serves on the Mathematics and Science Advisory Council for the Massachusetts Board of Education.

Dr. Chen provides high quality professional development in mathematics and science to teachers at all levels in Intensive Immersion Institutes. He works with school districts and school administrators to increase their capacity to support excellent mathematics and science instruction. He also works with higher education institutions to develop rigorous and effective pre-service and in-service preparation in mathematics and science. He leads a group working closely with teachers and college professors to develop CLEAR Math, intelligent courseware now in use with very positive outcomes in more than 35 school districts in Massachusetts.

Dr. Chen continues to teach and do research in physics, He received a BA in physics from National Taiwan University, and a PhD in physics from Columbia University.

MIKHAIL GOLDENBERG, PhD

Mikhail Goldenberg graduated from Odessa State University in 1961 with a master's degree in mathematics and mathematics education. He was a middle school and high school mathematics teacher for three years in Ukraine. He then moved to Russia where he received his PhD in Mathematics (Group Theory) in 1970 from Ural State University (Ekaterinburg). For many years (1964-1997) he was a professor of mathematics in South Ural State University (Chelyabinsk, Russia). He has worked with advanced high school students (Chelyabinsk Litseum) and mathematics teachers (Institute for Teachers Advance).

Dr. Goldenberg came to the United States in 1997 and became a mathematics teacher for the Ingenuity Project sponsored by The Abell Foundation. He is now the mathematics department head and teaches all the high school mathematics courses. He has led the Ingenuity Math Club for 10 years, and is a part-time lecturer at Morgan State University.

ROGER HOWE, PhD

Roger Howe has been teaching and conducting research in the Mathematics Department at Yale University for over 30 years. He is currently the William Kenan Jr. Professor of Mathematics. His mathematical research concerns symmetry and its applications. He has held visiting positions at many universities and research institutes in the U.S., Europe and Asia. He is a member of the American Academy of Arts and Sciences and the National Academy of Sciences.

Dr. Howe devotes substantial attention to issues of mathematics education. He has served on a multitude of committees, including those for several of the major reports on mathematics education of the past decade. He has reviewed mathematics texts and other instructional materials at all levels, from first grade through college. He has served as a member and as chair of the Committee on Education of the American Mathematical Society. He served on the Steering Committee of the Institute of Advanced Study/Park City Mathematics Institute, and has helped to organize a series of meetings at Park City devoted to increasing the contribution of mathematicians in mathematics education, especially refining understanding of the mathematical issues in K-12 mathematics curricula. He is currently a member of the U.S. National Committee on Mathematics Instruction. In 2006, he received the Award for Distinguished Public Service from the American Mathematical Society.

JASON KAMRAS

In April of 2005, President Bush named Jason Kamras, a 7th and 8th grade mathematics teacher at John Philip Sousa Middle School in the District of Columbia, the 2005 National Teacher of the Year. Mr.Kamras was recognized for helping his students make historic achievement gains in one of America's most disadvantaged communities. He was also recognized for cofounding and directing the EXPOSE digital photography program at his school, for which he received the Mayor's Art Award.

Mr. Kamras graduated summa cum laude from Princeton University in 1995, earning his bachelor's degree in public policy. He began teaching in 1996 as a member of Teach For America. In 2000, Mr. Kamras earned his master's degree from the Harvard Graduate School of Education.

Mr. Kamras currently works as Director of Human Capital Strategy in the Office of the Chancellor, District of Columbia Public Schools.

R. JAMES MILGRAM, PhD

Dr. Milgram is an emeritus professor of mathematics at Stanford University where he has taught since 1970. He is a member of the National Board of Education Sciences — the presidential board that oversees the Institute for Education Research at the U.S. Department of Education. He is also a member of the NASA Advisory Council, and is a member of the Achieve Mathematics Advisory Panel as well as a number of other advisory boards. He was one of the members of the Common Ground Project that included Deborah Loewenberg Ball, Joan Ferrini-Mundy, Jeremy Kilpatrick, Richard Schaar, and Wilfried Schmid.

From 2002 to 2005, Dr. Milgram headed a project funded by the U.S. Department of Education that identified and described the key mathematics that K-8 teachers need to know. He also helped to direct a project partially funded by the Thomas B. Fordham Foundation that evaluated state mathematics assessments. He is one of the four main authors of the California mathematics standards, as well as one of the two main authors of the California Mathematics Framework. He is one of the main authors of the new Michigan and Georgia K-8 mathematics standards.

Among other honors, Dr. Milgram has held the Gauss Professorship at the University of Goettingen and the Regents Professorship at the University of New Mexico. He has published over 100 research papers and four books, as well as serving as an editor of many others. His main area of research is algebraic and geometric topology, and he currently works on questions in robotics and protein folding. He received his undergraduate and master's degrees in mathematics from the University of Chicago, and his PhD in mathematics from the University of Minnesota.

ROBIN RAMOS

Robin Ramos is a mathematics coach at Ramona Elementary School in the Los Angeles Unified School District. She received her Bachelor of Arts degree at Northwestern University and her Masters at Mount St. Mary's College in Los Angeles. Before becoming a "math coach," she taught elementary school for 14 years.

Ms. Ramos developed effective instructional strategies by wide reading, training with Yoram Sagher in the use of Singapore mathematics curriculum materials, training as a mathematics coach, on-going collaboration with teachers, and continued work in classrooms. The community of Ramona Elementary, with a student body in which 94 percent of students are economically disadvantaged and 89 percent are second language learners, is very gratified by the great success students have shown on California's state mathematics assessments. Working intensively at one school site for many years, she has an appreciation of the daily challenges of the classroom.

YORAM SAGHER, PhD

Dr. Sagher is professor of mathematics at Florida Atlantic University and emeritus professor of mathematics at the University of Illinois, Chicago. He has written more than 55 research papers in Harmonic Analysis, Real Analysis, and Interpolation Theory. He has also written three research papers in mathematics education. Dr. Sagher directed ten doctoral dissertations in mathematics and one in mathematics education.

Dr. Sagher co-organized two international conferences in mathematics education: Numeracy and Beyond I, Pacific Institute for the Mathematical Sciences at the University of British Columbia, Vancouver, Canada, July 2003, and a follow-up conference, Numeracy and Beyond II, Banff, Canada, December 2004.

Dr. Sagher taught numerous continuing education courses for in-service elementary school and high school teachers in Chicago. He also created the course "Methods of Teaching High School Mathematics" at the University of Illinois, Chicago. The course serves as the capstone course for students preparing to become high school mathematics teachers.

Dr. Sagher developed highly effective teaching methods that, in combination with the Singapore mathematics textbooks, have produced outstanding results in elementary and middle schools from Boston to Los Angeles, including The Ingenuity Project in Baltimore and Ramona Elementary in Los Angeles.

Dr. Sagher is also interested in remedial mathematics education at the college level. He directed the doctoral dissertation of M.V. Siadat: "Building Study and Work Skills in a College Mathematics Classroom." For his work implementing the methods developed in that paper, Dr. Siadat was named "Illinois Professor of the Year" in 2005 by the Carnegie Foundation.

Dr. Sagher received his BS degree from the Technion, Israel Institute of Technology, and his PhD from the University of Chicago.

APPENDIX B: RATING PROGRAMS

SCORES FOR MATHEMATICS CONTENT INSTRUCTION

The instructional score for each program comprises equally weighted textbook and syllabi scores. Both textbooks and syllabi were evaluated on the basis of their coverage of the 12 essential topics. These topics, along with related subtopics, are outlined and elaborated on (see italicized font) below.

ESSENTIAL MATHEMATICS TOPICS

NUMBERS AND OPERATIONS

Topic 1: Whole numbers

1. Counting; numeration; the place-value system and its use in standard algorithms:
   Counting, ordering; definition of whole number; whole numbers represented by words, diagrams, symbols; definition of place value, the origin of the decimal system, values of places in decimals and powers of ten, saying decimal numbers and writing numbers with words; the meaning of addition, subtraction, multiplication and division with whole numbers in the context of our decimal place value system.
2. The four basic operations, their meaning and properties; computational methods in a decimal system:
   Why standard algorithms for adding and subtracting decimal numbers work; the commutative, associative, and distributive properties as they relate to operations; what is an algorithm; the addition, subtraction, multiplication, division algorithms.
3. Prime and composite numbers; the Fundamental Theorem of Arithmetic:
   Odd and even numbers, factors and multiples; divisibility tests.

Topic 2: Fractions and Integers

1. Fractions and their properties:
   Fractions represented by words, diagrams, symbols; modeling fractions as parts of a whole or as a count of a subset; placing fractions on a number line; equivalent fractions; comparing fractions; interpreting a fraction as division; common denominators; simplest form; mixed numbers and improper fractions.
2. The four basic operations on fractions:
   Adding and subtracting with like and unlike denominators; the meaning of multiplication for fractions; the procedures for multiplying fractions; mixed number answers to whole number division problems; using division to convert improper fractions to mixed numbers; interpreting division for fractions; the "invert and multiply' procedure for division.
3. Basic operations on positive and negative numbers.

Topic 3: Decimals

1. Computations with decimals:
   Decimals represented by words, diagrams, symbols; representing decimals, numbers on number lines, comparing sizes of decimal numbers; explaining the shifting of decimal points.
2. Decimals and common fractions; ratio, proportion, percent:
   Decimal representations of fractions; ratios and fractions; equivalent ratios; solving proportions; using proportions; the meaning of percent; the three types of percent problems; percent increase and decrease; adding percentages.
3. Real numbers and the number line:
   Rational and irrational numbers; relationships among number systems.

Topic 4: Estimation

Estimating results of computations; estimating measurements; how to round.

ALGEBRA

Topic 5: Constants and variables; writing and reading algebraic expressions, including those with parentheses

Letters; numerical expressions, algebraic expressions; equations; symbolic manipulation.

1. Powers and exponents; properties of powers with integer and rational exponents:
   Powers of 10; powers of numbers other than 10; scientific notation.
2. Monomials and polynomials; adding, subtracting, multiplying and dividing polynomials.
3. Relationships among variables; formulas and functions:
   Pairs of numbers following a given rule; finding rules for relations when given pairs of numbers.

Topic 6: Equations

1. Evaluating algebraic expressions; identities and the equation:
   Symbolic manipulation.
2. Solving linear equations:
   Solving equations by isolating variables.

Topic 7: Graphs and functions

1. The Cartesian plane; graphing a function; graphing linear equations in two variables.
2. Solving systems of two linear equations in two variables.

GEOMETRY AND MEASUREMENT

Topic 8: Measurement and units of measurement

The concept of measurement; standard and non-standard units; systems of measurement; error and accuracy; length, area, volume, dimension; converting from one unit of measurement to another.

Topic 9: Basic concepts of plane geometry

1. Lines, rays, segments; measuring segments; angles and angle relationships; measuring angles:
   Planes; parallel and perpendicular lines.
2. Geometric figures: congruency, similarity, symmetry, scale factors, auxiliary lines.
3. Inductive and deductive reasoning; proof.

Topic 10: Polygons and circles

1. Triangles, right triangles, the Pythagorean theorem.
2. Quadrilaterals and their properties:
   Showing relationships with Venn diagrams.
3. The circle and the arc of the circle; measure of a central angle; chords; angles subtended by cords.

Topic 11: Perimeter and area; surface area and volume

1. Perimeter of a polygon; area formulas for rectangles and triangles.
2. Circumference and area of a circle:
   Pi.
3. Simple solids; volume formulas for cuboids and cylinders:
   Areas, volumes and scaling.

DATA ANALYSIS AND PROBABILITY

Topic 12: Probability and data characteristics

1. Drawing and interpreting graphs, tables, bar graphs, pie charts.
2. Data characteristics:
   Range, mean, median.
3. Frequency and probability:
   Simple probability rules.

SCORING ELEMENTARY MATHEMATICS TEXTBOOKS

Each textbook was evaluated by mathematicians on the NCTQ Mathematics Advisory Group on the basis of its treatment of the 12 essential mathematics topics and their subtopics.

These mathematicians evaluated the adequacy of treatment of subtopics using the rubric below:

A. Depth:	B. Connection:	C. Integrity:	D. Examples:	E. Methods:
Coverage adequate considering topic and audience.	Explicit conceptual connections made to enhance understanding.	Exposition not mechanical and illustrates mathematical reasoning process.	Sufficient and selected for maximum pedagogical significance.	Connection between the math content and the way that content should be delivered.

A subtopic could receive a score of "absent," "0," "1," or "2" in each of these five areas (a through e), with "0" representing a deficiency, "1" representing adequacy, and "2" representing excellence.

SCORING ELEMENTARY CONTENT COURSE SYLLABI

A course or set of courses could be awarded up to 6 points in each of the 12 essential math topics, for a percentage score out of 72 total points. The points could be awarded for one course or divided among different courses in one education school. For example, if one elementary content course dealt with the Cartesian plane and a second dealt with graphing functions and graphing linear equations in two variables, the total of 6 points for that algebra topic would be awarded for the program. If, on the other hand, both courses dealt with the Cartesian plane and graphing functions and linear equations in two variables, a total of 6 points would still be awarded.

See Appendix E for samples of syllabi that indicate how points might be awarded in evaluation.

FACTORING CONSIDERATION OF DEPTH INTO THE INSTRUCTIONAL SCORE

The instructional score that emerges from this process of scoring textbooks and syllabi does not take into consideration the time spent on instruction, i.e., the "depth" of the course. A program that uses an excellent textbook in a single course that addresses all 12 essential topics may have a very high score, irrespective of the fact that adequate time could not possibly be spent on the 12 topics in only one course. For that reason, the number of courses in a program is used to refine the instructional score.

Except in the cases of the "most selective" schools, where the Mathematics Advisory Group felt that prospective teachers would need only six semester credits to cover the requisite material, the ***final instructional score*** was the same as the instructional score if the education program had the equivalent of nine semester credits of elementary content coursework. Otherwise, the instructional score was reduced proportionally to generate a final score reflecting lesser amounts of time devoted to elementary content instruction.[1]

The final instructional score was used in the statistical analysis of whether institutional characteristics explained score differences among education schools.

CATEGORIZING EDUCATION SCHOOLS

Those education schools with instructional scores of 70 percent or above that required nine semester credits of elementary content instruction (six in the case of a "most selective school") were identified as passing or "schools with the right stuff."

Those education schools with instructional scores of 70 percent or more that required fewer than nine semester credits of elementary content instruction were identified as "schools that would pass if more coursework was required."

Those education schools with instructional scores of less than 70 percent that required nine or more[2] semester credits of elementary content instruction were identified as "schools that would pass with better focus and textbooks."

Those education schools with instructional scores of less than 70 percent that required fewer than nine semester credits of elementary content instruction were identified as "schools that fail on all measures."

1 An instructional score might also be adjusted upward by half of the percentage (up to 10 percent) of time allocated to "non-essential topics." This adjustment was required to compensate for the fact that courses in which nonessential topics were addressed received a score of "zero" in their textbook rating for the percentage of coverage of nonessential topics and textbook ratings comprised half of the instructional score. Because up to 15 percent of a program's coursework could address nonessential topics and still be deemed in our evaluation to properly allocate instruction to the four critical areas in elementary mathematics preparation, this adjustment — effectively allowing up to 20 percent of coverage to be nonessential without penalty — is a generous boost to many programs that devoted excessive time to nonessential topics.

2 Two school in this category require only eight credits of elementary content coursework.

APPENDIX C: CONSENSUS ON RECOMMENDED MATHEMATICS TOPICS THAT ELEMENTARY TEACHERS MUST UNDERSTAND

NCTQ MATH STUDY (Pre-K to 8th Grade)	**NCTM FOCAL POINTS CONTENT STRANDS** (Pre-K to 8th Grade)	**TIMMS STUDY (2003)** (Grades 4th and 8th)
Numbers and Operations	**Numbers and Operations**	**Numbers and Operations**
TOPIC: Whole numbers Subtopics: ■ Counting, numeration, the place value system and its use in standard algorithms ■ The four basic operations: their meaning and properties; computational methods in a decimal system ■ Prime and composite numbers; the Fundamental Theorem of Arithmetic	TOPIC: Whole numbers Subtopics: ■ Counting, comparing ■ Addition and subtraction ■ Multiplication and division ■ Place value structure; place after counting ■ Primes and factorization	TOPIC: Whole numbers Subtopics: ■ Whole numbers, including place value and ordering ■ Represent whole numbers using words, diagrams, or symbols ■ Properties of whole numbers, such as odd/even, multiples, or factors ■ Operations, computation ■ Place value factorization operations
TOPIC: Fractions and integers Subtopics: ■ Fractions and their properties ■ Operations on fractions ■ Operations on integers	TOPIC: Fractions and integers Subtopics: ■ Representing; operations with integers	TOPIC: Fractions and Integers Subtopics: ■ Equivalent, common, computations ■ Integers, including words, numbers, or models ■ Adding/subtracting fractions with the same denominator ■ Compare and order fractions
TOPIC: Decimals Subtopics: ■ Operations with decimals ■ Decimals and common fractions, ratio, proportion, percent ■ Real numbers and the number line	TOPIC: Decimals Subtopics: ■ Multiplication and division, including decimals ■ Representing and equivalence, including decimals; ratio and proportion ■ Addition and subtraction, including decimals	TOPIC: Decimals Subtopics: ■ Fractions or decimals represented by words, numbers, or models ■ Computations with decimals ■ Conversion of percents to fractions or decimals and vice versa ■ Ratios; simple and proportional reasoning

NCTQ MATH STUDY (Pre-K to 8th Grade)	NCTM FOCAL POINTS CONTENT STRANDS (Pre-K to 8th Grade)	TIMMS STUDY (2003) (Grades 4th and 8th)
Numbers and Operations (cont)	**Numbers and Operations** (cont)	**Numbers and Operations** (cont)
TOPIC: Estimation and rounding Subtopics: ■ Estimating results of computations ■ Estimating measurements ■ How to round	TOPIC: Estimation and rounding	TOPIC: Estimation and rounding Subtopics: ■ Estimations, approximations with whole numbers
Algebra	**Algebra**	**Algebra**
TOPIC: Constants and variables; writing and reading expressions Subtopics: ■ Powers with integer and rational exponents ■ Operations with monomials and polynomials ■ Relationships among variables, formulas, functions	TOPIC: Constants and variables; writing and reading expressions Subtopics: ■ Variables as symbols ■ Geometric and number patterns ■ Commutative, associative and distributive properties ■ Linear functions	TOPIC: Constants and variables; writing and reading expressions Subtopics: ■ Numeric, algebraic, and geometric patterns or sequences ■ Sums, products and powers of expressions containing variables ■ Finding a rule for a relation given some pairs of numbers ■ Pairs of numbers following a given rule ■ Two-variable equations ■ Equivalent representations of functions as ordered pairs, tables, graphs, words, or equations ■ Proportional, linear, and nonlinear relationships
TOPIC: Equations Subtopics: ■ Evaluating expressions, identities and equations ■ Solving equations	TOPIC: Equations Subtopics: ■ Expressions and equations ■ Solving linear equations	TOPIC: Equations Subtopics: ■ Equality using equations, areas, volumes, masses/weight ■ Missing numbers in an equation ■ Simple linear equations and equalities
TOPIC: Graphs and functions Subtopics: ■ The Cartesian plane; graphing a function, graphing linear equations in two variables ■ Solving systems of two linear equations in two variables	TOPIC: Graphs and functions Subtopics: ■ Graphs and tables ■ Graphing linear equations	TOPIC: Graphs and functions Subtopics: ■ Attributes of a graph ■ Cartesian plane: ordered pairs, equations, intercepts, intersections, and gradient (in Geometry)

NCTQ MATH STUDY (Pre-K to 8th Grade)	**NCTM FOCAL POINTS CONTENT STRANDS** (Pre-K to 8th Grade)	**TIMMS STUDY (2003)** (Grades 4th and 8th)
Geometry and Measurement	**Geometry and Measurement**	**Geometry and Measurement**
TOPIC: Basic concepts of plane geometry Subtopics: ■ Lines, rays, segments; angles, angle measurement and relationships ■ Geometric figures: congruency, similarity, symmetry, auxiliary lines, scale factors ■ Inductive and deductive reasoning; proof	TOPIC: Basic concepts of plane geometry Subtopics: ■ Lines and angles ■ Symmetry ■ Congruence and similarity	TOPIC: Basic concepts of plane geometry Subtopics: ■ Angles greater than, equal to, or less than a right angle; acute, right, straight, obtuse, reflex, complementary, supplementary ■ Properties of angles bisectors and perpendicular bisectors of lines ■ Parallel and perpendicular lines ■ Relationships of angles at a point, angles on a line, vertical angles, angles formed by transversal and parallel lines, perpendicularity ■ Points in a plane ■ Symmetry about a line ■ Rotational symmetry for 2-dimensional shapes ■ Two-dimensional symmetrical figures
TOPIC: Polygons and circles Subtopics: ■ Triangles, right triangles, Pythagorean theorem ■ Quadrilaterals and their properties ■ Circles: arcs, central angles, chords	TOPIC: Polygons and circles Subtopics: ■ Identifying and describing shapes ■ Composing and decomposing shapes ■ Describing and analyzing properties of two-dimensional shapes	TOPIC: Polygons and circles Subtopics: ■ Familiar 2- and 3-dimensional shapes and their properties ■ Congruent and similar triangles; congruent figures and their corresponding measures ■ Pythagorean theorem to find the length of a side ■ Construct or draw triangles and rectangles of given dimensions
TOPIC: Perimeter and area, surface area, volume Subtopics: ■ Perimeter and circumference; area formulas ■ Simple solids; volume formulas	TOPIC: Perimeter and area, surface area, volume Subtopics: ■ Perimeter and area ■ Surface area and volumes	TOPIC: Perimeter and area, surface area, volume Subtopics: ■ Relationships between 2- and 3-dimensional shapes ■ Measurement formulas for perimeter of a rectangle, circumference, area of plane figures, surface area and volume of rectangular solids and rates ■ Measure of irregular or compound areas

NCTQ MATH STUDY (Pre-K to 8th Grade)	NCTM FOCAL POINTS CONTENT STRANDS (Pre-K to 8th Grade)	TIMMS STUDY (2003) (Grades 4th and 8th)
Geometry and Measurement (cont)	**Geometry and Measurement** (cont)	**Geometry and Measurement** (cont)
TOPIC: Measurement Subtopics: ■ The concept of measurement ■ Standard and non-standard units ■ Systems of measurement ■ Error and accuracy ■ Length, area, volume, dimension ■ Converting from one unit of measurement to another	TOPIC: Measurement Subtopics: ■ Length, capacity, weight	TOPIC: Measurement Subtopics: ■ Nonstandard units to measure lengths, area, volume, and time ■ Standard units to measure length, area, volume, perimeter, circumference, speed, density, mass/weight, angle and time ■ Instruments to measure length, area, mass/weight, angle, speed, angle, and time ■ Conversion factors between standard units; relationships among units of conversions within systems of units and for rates ■ Estimating length, circumference, area, volume, weight, time, angle and a speed in problem situations ■ Computations with measurements in problem situations ■ Precision of measurements
TOPIC: Other	TOPIC: Other	TOPIC: Other Subtopics: ■ Informal coordinate systems ■ Translation, reflection, rotation, and enlargement

NCTQ MATH STUDY (Pre-K to 8th Grade)	NCTM FOCAL POINTS CONTENT STRANDS (Pre-K to 8th Grade)	TIMMS STUDY (2003) (Grades 4th and 8th)
Data Analysis and Probability	**Data Analysis and Probability**	**Data Analysis and Probability**
TOPIC: Probability and data characteristics Subtopics: ■ Drawing and interpreting graphs, tables, bar graphs, pie charts ■ Data characteristics (range, mean, median) ■ Frequency and probability; simple probability rules	TOPIC: Probability and data characteristics Subtopics: ■ Interpreting graphical representations of data ■ Mean, median, range ■ Probability	TOPIC: Probability and data characteristics Subtopics: ■ Simple probability, including using data from experiments to estimate probabilities for favorable outcomes ■ Recognizing what various numbers, symbols and points mean in a data display ■ Organizing a set of data by one or more characteristic using tally chart, table, or graph ■ Reading from, displaying data using, and interpreting tables, pictographs, graphs, and charts ■ Comparing and matching different representations of the same data ■ Characteristics of related data sets ■ Drawing conclusions from data displays ■ Data collection methods ■ Sources of error in collecting and organizing data ■ Characteristics of data sets, including mean, median, range and shape of distribution ■ Interpreting datasets ■ Evaluating interpretations of data with respect to correctness and completeness of interpretation

APPENDIX D: RATING OF ELEMENTARY CONTENT MATHEMATICS TEXTBOOKS

TEXTBOOK SCORES

The following table summarizes the scores of all textbooks that received two reviews:

Author and Textbook	Numbers & Operations (N&O) (54 points possible)	Bonus points for excellence in N&O	Algebra (39 points possible)	Bonus points for excellence in Algebra	Geometry (54 points possible)	Bonus points for excellence in Geometry	Data Analysis & Probability (19 points possible)	Total Score
Bassarear *Mathematics for Elementary School Teachers*	**21 (low)**	2	**3 (low)**	0	**33**	17	**19**	**76**
Beckmann *Mathematics for Elementary Teachers*	**54 (high)**	18	**29**	21	**48**	11	**19**	**150 (high)**
Bennett, Nelson *Mathematics for Elementary Teachers: A Conceptual Approach*	**33**	4	**15**	0	**41**	17	**19**	**108**
Billstein, Libeskind, Lott *A Problem Solving Approach to Mathematics for Elementary School Teachers*	**35**	6	**38 (high)**	18	**50**	15	**19**	**142**
Long, DeTemple *Mathematical Reasoning for Elementary Teachers*	**29**	3	**5**	0	**47**	13	**19**	**100**
Miller, Heeren, Hornsby *Mathematical Ideas*	**23**	0	**19**	0	**7 (low)**	0	**19**	**68 (low)**
Musser, Burger, Peterson *Mathematics for Elementary Teachers: A Contemporary Approach*	**45**	12	**16**	0	**45**	11	**19**	**125**
O'Daffer, et al. *Mathematics for Elementary School Teachers*	**36**	0	**5**	0	**44**	0	**19**	**104**
Parker, Baldridge *Elementary Mathematics for Teachers* and *Elementary Geometry for Teachers*	**54 (high)**	50	**24**	20	**54 (high)**	49	**NA**[1]	**132**
Sharhangi *Elements of Geometry for Teachers*[2]	**NA**	0	**NA**	0	**16**	0	**NA**	**(16)**

1 A new edition of *Elementary Geometry for Teachers* will include a section on data analysis and probability.

2 Used in one course in our sample.

Author and Textbook	Numbers & Operations (N&O) (54 points possible)	Bonus points for excellence in N&O	Algebra (39 points possible)	Bonus points for excellence in Algebra	Geometry (54 points possible)	Bonus points for excellence in Geometry	Data Analysis & Probability (19 points possible)	Total Score
Sonnabend *Mathematics for Teachers: An Interactive Approach for Grades K-8*	**33**	2	**(0[3])**	0	**44**	16	**19**	**96**
Sowder, Sowder, Nickerson *Reconceptualizng Mathematics*	**23**	2	**9**	2	**30**	0	**19**	**81**
Van de Walle[4] *Elementary and Middle School Mathematics: Teaching Developmentally*	**16**	0	**2**	0	**5**	0	**11**	**34**

The following textbooks were reviewed only once. Each was used in only one course in our sample:

Bennet, Briggs, Triola, *Statistical Reasoning for Everyday Life*
(Elementary content course; adequate in data analysis and probability, the only critical area covered.)

Burger, Starbird, *The Heart of Mathematics: An Invitation to Effective Thinking*
(Elementary content course: inadequate in all critical areas.)

Chapin, Johnson, *Math Matters: Understanding the Math You Teach Grades K-8*
(Mathematics methods course; adequate only as a supplemental textbook.)

Heddens, Speer, *Today's Mathematics: Concepts, Classroom Methods, and Instructional Activities*
(Elementary content course: adequate only in data analysis and probability.)

Kaplan, *Math on Call: A Mathematics Handbook*
(Elementary content course; inadequate in all critical areas)

Kutz, Lubell, Burns, *Foundations of Mathematics II*
(Elementary content course: covers all critical areas but data analysis; inadequate in all areas.)

See Appendix B for a discussion of the rubric for and method of scoring textbooks.

3 This section deemed inadequate on initial review and because the textbook was used in fewer than five courses the section was not reviewed twice.

4 Methods textbook evaluated for content. See also evaluation for methods in Appendix F.

COMMENTS ON TEXTBOOKS

The following excerpts from textbook evaluations provide more information on the mathematics content textbooks that received the highest and lowest overall scores, as well as the highest and lowest scores in the areas of numbers and operations, algebra, and geometry and measurement. (No textbook's data analysis and probability section received a score from a second review.)

Highest overall textbook score: Beckmann

While it has a number of minor flaws, this textbook is head-and-shoulders above the other books. **It received the highest overall rating of all the textbooks evaluated**. It is mathematically sound while still accessible for the audience, and has a very coherent structure. The problems in the text are excellent. This is one of the best textbooks for pre-service teachers and sets appropriately high standards for others.

An *Activities Manual* by the same author is often assigned in conjunction with this text. The manual is an exciting book of well-thought-out activities, some of which are very challenging. The reviewer considers this a "must-have" for the prospective teachers using the textbook.

Lowest overall textbook score: Miller

There is little in this text that is good. **It received the lowest overall rating of all the commonly used textbooks evaluated**. (One reviewer found it shocking that the text is now in its 11th edition and some of the worst parts seem not to have ever been noted and corrected by the authors.) The objective of the book is unclear; it appears to be list of disjointed topics, with quite a few "interesting" diversions like Fibonacci numbers, graph theory, even group theory. However, nothing is treated in any depth, and nothing important is presented in a way that would translate into the classroom.

NUMBERS AND OPERATIONS:

Highest numbers and operations score: Beckmann, and Parker and Baldridge

Beckmann

This textbook was tied for the highest score of all textbooks reviewed for its numbers and operations section. One of the reasons is that its discussion of real numbers, finite and ultimately repeating decimals and how to write them as fractions, as well as the converse, is exemplary. There is also a rock-solid discussion of square roots and their irrationality.

Parker and Baldridge

This textbook covers primarily numbers and operations, although it does contain a pre-algebra section. **It tied for the highest rating of all textbooks for its coverage of numbers and operations, and had the highest number of bonus points for excellence in this area**. A few excerpts from its review may explain why:

- In addition to the catchy Ratio and Proportion word problems, which are optimized to be solved by the model drawing technique (e.g. Example 2.4 on page 174), the text contains other challenging yet age-appropriate word problems (e.g., #9, #10 on p. 130; #9 on P. 184)

- The processes of counting up and counting down are explained in intuitive ways, justifying the reasoning behind the processes. (p.21). This is good information for teachers.
- Connections to the classroom are explicit. For example, the timeline demonstrating the expected lag of subtraction with respect to addition is very relevant information. The authors did a thoughtful job here and in many other places.
- Rule 1 on page 187 (regarding signed numbers) is an essential, classic case requiring deep understanding. Most pre-service texts do not go this far-- i.e., dealing with generic symbols of any kind. Demanding students to think beyond numerical examples early on is important for topics such as signed numbers. This approach is sound and rewarding in the long run.

The "pre-algebra" section is well done and its strategic placement is very significant: The basic concepts and ideas of pre-algebra can and should be exposed foundationally before rational and real numbers enter the stage because the subtleties related to the new number species can potentially complicate the basic laws of operations for first-time learners.

While the text is most powerful when used in conjunction with the Singapore primary math textbooks, those using it "stand-alone" are only slightly handicapped.

Lowest numbers and operations score: Bassarear

This very widely used textbook received the lowest rating in numbers and operations of any of the commonly used textbooks we evaluated. The treatment of numbers and operations had only one bright spot: estimation. Otherwise, the review of this text was a litany of inadequacies. This excerpt from the reviewer's comments will give a flavor of the critique:

> *On page 110, after a discussion of other systems of numeration, it becomes clear that the author does not quite understand that base ten place value is a way of representing the expanded form using a minimum number of symbols. Rather, for him, the expanded form is just one way of representing the base 10 number. As a result, it is only given a half page, and the discussion is incredibly superficial.*
>
> *In the next chapter, the four basic operations are discussed. The treatment of addition is discursive, assuming that the readers already know what it is. Everything is developed in what we know are the worst ways for optimizing learning in young students – especially weak ones. As an example, addition is properly defined on pg. 125, but then IT IS NOT SHOWN that the definition makes sense, i.e. that the sum is independent of which disjoint sets are combined. So we have three crucial lines of definition and go back to models. The same happens with subtraction on page 149. Three lines of definition are given, and then discussion moves to other things. The fact that subtraction is addition in disguise is not used again. This same pattern holds for multiplication and division.*

In general, while it is apparent that the author tried very hard to simplify the content, and the colloquial writing style makes the book very easy to read, this is not a textbook in the traditional sense. The depth and coverage are insufficient on many topics and the low expectations of readers make it inadequate as a textbook for content-based courses. One reviewer stated: "By the time I finished the book I was genuinely horrified. No wonder our teachers have such problems with the subject!"

This textbook is often assigned with an accompanying *Explorations* textbook written by the same author. One reviewer commented: "There is no substance in this 'explorations' book. All the activities are low-level skill practices blended with an unnecessary game-motif. The explorations supplement has no educational value at the college pre-service level. It should receive an absolute failing grade."

ALGEBRA:

Highest algebra score: Billstein

This textbook received the highest rating in algebra of the textbooks evaluated. While this may seem counterintuitive in a book that doesn't even have chapters explicitly dedicated to algebra, important differentiating features include very explicit sections and language on bridging arithmetic and algebra, and early exposure of function and set concepts.

In general, this textbook is far better than average. There is enough on mathematical reasoning and many problems. (Too many in a sense, since the best ones might be omitted in assignment.) The "Sample School Book Page" is implemented well, and "Questions From the Classroom" are very relevant.

Lowest algebra score: Bassarear

This very widely used textbook received the lowest rating in algebra of any of the commonly used textbooks we evaluated. The author mentions in the preface that the text will de-emphasize algebraic procedures and focus on "algebraic thinking." But the material covered in the text is no more than basic pattern recognition and it does not even go very far with that. Chapter 1 of the book contains many "cute" problems and profound points. In and of itself, the chapter is an interesting piece of work and could form the core of a good course. But the rest of the algebra discussion is less than "math appreciation." It talks about algebra rather than DOES algebra. The problems are anemic — in the name of doing "explorations."

GEOMETRY AND MEASUREMENT:

Highest geometry and measurement score: Parker and Baldridge

The quality of this geometry textbook is very high. **It received the highest rating for its coverage of geometry of all the textbooks reviewed**. The mathematical reasoning is correct and found throughout, yet the explanations seem simpler than in other texts. It also contains many excellent problems, especially multistep problems.

This text correctly prioritizes what is important in geometry and what is not. It also elegantly lays the groundwork for more advanced mathematics. For example, the finding-unknown-angle problems not only contain the essence of a number of general theorems in geometry, they also help students learn how to set up and solve linear equations.

The reviewer noted with approval the manner in which concepts that might seem daunting are made inviting and accessible. For example, formulas for the volume or the surface area of a sphere are introduced to prospective elementary teachers as the culmination of the K-8 measurement curriculum: These are amazing formulas! They are subtle, yet are simple to learn and to use, and they have important applications in science. This section shows how these formulas can be explained in terms of elementary mathematics.

The text distinguishes between "teacher solutions" and "student solutions," useful in training teachers so that they will perceive how the level of knowledge that they must have differs from that of their students.

Lowest geometry score: Miller

This textbook received the lowest rating in geometry of any of the commonly used textbooks evaluated. In general, its treatment of topics simply did not rise to the level of adequacy. There are many places where mathematical reasoning could have been given, but was not. Many problems are inappropriately set at the level of late elementary school.

The basic approach taken seems to be that if something "looks" like it is true, it should be taken as true by the reader, unless the author's purpose is to prove it. Likewise, formulas are provided or used without derivation, inappropriate for a textbook for teachers.

APPENDIX E: SAMPLE ELEMENTARY MATHEMATICS COURSE SYLLABI AND HOW THEY WERE SCORED

SYLLABUS FOR A COURSE WITH HIGH SCORES IN THE THREE CRITICAL AREAS COVERED: NUMBERS AND OPERATIONS, ALGEBRA, AND GEOMETRY

specific, relevant

COURSE DESCRIPTION:

This course is intended to provide elementary education majors with experiences in becoming independent problem solvers while providing a solid foundation for teaching early mathematics. Topics include set theory, systems of numeration, number theory, properties of while numbers, rational numbers, and real numbers, estimation, beginning geometry, and measurement. Collaborative learning, discovery, and refinement of presentation skills are stressed through in-class experiences. Traditional mathematical content is covered in the context of developing student competence with respect to the abilities outlined in the process standards found in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000).

COURSE OUTCOMES:

Students will demonstrate critical reasoning skills and problem solving competency in the following areas: set theory, systems of numeration, number theory, properties of whole numbers, rational numbers, and real numbers, estimation, beginning geometry, and measurement.

Students will develop competence with respect to the five process standards found in *Principles and Standards for School Mathematics* published by the National Council of Teachers of Mathematics in 2000:

1. Problem solving-students will become more confident and independent problem solves
2. Reasoning and proof-the student's ability to use deductive, inductive, and intuitive reasoning will grow, and she will be able to explain her solution process
3. Communication-students will appreciate the role of discussion in learning mathematics and the value of notation and vocabulary as precise tools that make communication easier
4. Connections-students will become more aware of connections between various mathematical topics and of connection between mathematics and many other application areas
5. Representation-the student will increase her ability to create and use mathematical representations to model and interpret mathematical ideas and concepts

Students will not only examine traditional mathematical content at the level at which they will be teaching but also at a deeper level (an "adult level perspective") so that they will be able to teach from a full understanding of the content (from a so-called "overflow of knowledge") and thus be able to examine topics from many different perspectives and appreciate multiple strategies.

INSTRUCTION METHODS:

The primary method of instruction will be lectures and discussions supported heavily by homework assignments. The homework will consist of pencil and paper problems as will as problems to be solved via the computer. One of the most effective ways to learn mathematics is through practice and individual exploration; thus, the course is heavily homework intensive. Daily homework problems will be assigned, and the student is expected to have completed these problems before the next class meeting and be prepared to share in class discussions relating to these assignments. Specific homework assignments will be collected and graded regularly. Active individual and small group class participation, sharing, and involvement will be expected and encouraged. The student should consult the instructor with any questions/difficulties encountered in her/his studies; a student may be referred to the advising center for additional assistance. *Students with documented disabilities who may need academic accommodations should discuss these needs with the instructor during the fist two weeks of class. Students with disabilities who wish to request accommodations should contact the Advising Center*.

CALCULATORS:

Technology is essential in teaching and learning mathematics, but it cannot be used as a replacement for basic understanding and intuition. The student will make use of calculators and the computer as the necessary tools to enhance student learning. A scientific calculator is necessary for this course — one that has, as a minimum, the usual arithmetic operations (+, -, x, / ^) in addition to memory keys.

EVALUATION:

Three in-class quizzes and a cumulative final exam will be given. Homework will be collected and graded regularly. Class participation and individual effort will also enter into the computation of the student's grade. Your obligations for this course include attendance at the final exam, on the day and time scheduled by the Registrar's Office. You should not make travel arrangements until the final exam schedule is published; if you must make plans early, you should schedule your travel after the last final exam day. Each student is expected to do her own work; do not invite trouble directly with someone else (unless specifically encouraged to collaborate) or by using materials not authorized by the instructor. Violations of the Honor Code will be handled by the instructor, will be reported to the Dean, and will result in a grade of zero on the assignment/exam.

Grades will be based on a relative scale with the following tentative weights:

Quizzes	45%	(15% each)
Final Exam	20%	
Homework	25%	
Instructor Evaluation	10%	(includes attendance, individual, and group participation)
	100%	

REQUIRED TEXTS:

Mathematical Reasoning for Elementary Teachers, fourth edition, Long, Calvin, and DeTemle Duane, Addison, Wesley, Longman Publishing, 2006.

25 not counting assessment days

TENTATIVE COURSE OUTLINE

Problem-solving should be in the context of elementary content problems not in the abstract: counts as "non-essential" topic.

Class Meeting		Topic
1/16		1.1 — introduction; principles of problem solving, guess and check
1/18		1.2 — solving problems using diagrams, lists, and tables
1/23	algebra	1.3 — searching for patterns, using variables, solving similar problems
1/25		1.4 — problem solving by working backwards, eliminating possibilities
1/30		Review
2/1		**Quiz #1**
2/6	Numbers and Operations (N&O)	2.1 — sets and set operations
2/8		2.2 — 1-to-1 correspondences, set equivalence
2/13		2.3, 2.4 — wholenumbers, operations with whole numbers
2/15	algebra	2.4 — exponents
2/20	N&O	3.1, 3.2 — numeration systems, bases other than base 10
2/22		3.3, 3.4 — algorithms for addition, subtraction, multiplication and division
2/27		**Quiz #2 (chapter 2, 3.1-3.2)**
3/1	N&O	3.5, 3.6 — estimation, mental math, calculator use
3/13		4.1, 4.2 — divisibility, prime and composite numbers
3/15		4.3 — greatest common factors, least common multiples
3/20		5.1, 5.2 — integers, integer addition, subtraction
3/22		5.3, 5.4 — integer multiplication, division
3/27		6.1, 6.2 — arithmetic of rational numbers
3/29		6.2, 6.3 — rational number system
4/3		**Quiz #3 (chapters 4, 5, and 6)**
4/5	N&O	7.1 — decimals
4/10		7.2, 7.3 — computation with decimals, ratio, proportion, and scale
4/12		7.4 — percent

All 24 N&O points awarded for 14 class sessions.

4/17 algebra 8.1-8.3 — algebraic equations, functions
4/19 11.1, 11.2 — basic geometric notions, polygons
4/24 geometry 12.1, 12.2 — measurement, area and perimeter
4/26 13 — symmetry

Four of 18 algebra points deducted for lack of coverage of polynomials (1 pt.) and systems of equations (3 pts.).

Five of 24 geometry points deducted for lack of coverage of congruence and similarity (1 pt.), covered in Chap. 14 of Lang), reasoning and proof in the context of geometry (1 pt.), failure to address Pythagorean theorem (1 pt., covered in Sec. 12.3 of Lang), and failure to address solids and volume (2 pts., covered in Sec. 12.4 of Lang).

Time Allocation based on the proportion of class sessions:

N&O: 55%
Algebra: 10%
Geometry: 10%
Data Analysis: 0%
Non-essentail topics: 25%

For more information on topics and subtopics in each of the four critical areas of mathematics, see Appendix B.

SYLLABUS FOR A COURSE WITH A LOW SCORE IN NUMBERS AND OPERATIONS

The intention of this course was to only cover "Numbers and Operations" so it was evaluated only on the coverage of the math topics covered in numbers and operations.

TEXTBOOK:

Billstein-Libeskind-Lott, *A Problem Solving Approach to Mathematics*, Ninth edition, Addison-Wesley Publishing Co., Inc. 2007.

CATALOG DESCRIPTION:

Axiomatic development of number system extension of the concept of number; basic operations of arithmetic with emphasis on use of axioms; sets and relations. Course is designed to equip students for teaching mathematics in elementary schools. Three hours credit.

PREREQUISITE:

Entrance requirements

ACT score of 18, SAT equivalent or completion of Math 098, 111 with a grade of "C" or better.

I. Purpose

A. To equip students with a working knowledge of these principles and methods which are basic to the teaching of mathematics in elementary school.

B. To develop in students the ability to think and work accurately in terms of quantitative relationships and the logic of the scientific method.

II. Objectives of the Course

A. General learning objectives

1. To acquaint the student with the complex number system.
2. To introduce the concept of base numbers other than base 10.
3. To introduce the different methods of presenting mathematical concepts to children in elementary school.
4. To acquaint the student with critical thinking and problem solving

B. Specific behavioral objectives

As a result of this course the student should be able to:

1. Solve problems using sets and logic notation.
2. Use the ideas of addition and subtraction of whole numbers in the union and intersection of sets.
3. Use number activities as a teaching aid.
4. Use the number line as a number activity for teaching the four basic arithmetic operations.
5. Convert numbers in other bases (such as base 2) to a corresponding number in base 10 and base 10 numbers to corresponding numbers in other bases.

6. Use many different algorithms for solving problems in addition, subtraction, multiplication, and division.
7. Identify components of a problem and apply problem-solving skills to finding a solution.

III. Course Calendar: The class will meet 150 minutes per week

IV. Topics to be Covered

Time Allocation: Of 26 topics 23-24 (including the use of frames and numbers activities and one set topic) address Numbers and Operations topics: 90%

A. Numbers and numerals
   1. Concept of a whole number
   2. Developing a number language
   3. Numerals in base 10
   4. Changing from one base to another
   5. Finite and infinite sets

B. Basic ideas of addition and subtraction of whole numbers
   1. The union and intersection of sets
   2. Definition of the addition of whole numbers
   3. Set complements
   4. Number lines
   5. The use of frames and number activities

Time Allocation: While sets are an important conceptual framework with which to consider numbers and operations, specifying three topics in sets (see A5 above) is disproportionate. Two are classified as "non-essential": 10%

C. Addition and subtraction algorithms for whole numbers
   1. Addition and subtraction with abacus and stick bundles.
   2. The addition and subtraction algorithm in symbols
   3. Addition and subtraction in other bases

D. Basic concepts of multiplication and division of whole numbers
   1. Multiplication in terms of Cartesian products of sets
   2. Division as partition and the inverse of multiplication
   3. Properties of multiplication and division
   4. Number activities involving multiplication and division
   5. Zero, the troublemaker

Twelve of 24 N&O points awarded

E. Multiplication and division algorithms for whole numbers
   1. Multiplication and division algorithms
   2. Multiplication and division in other bases
   3. The GCD and LCM

1 pt. deducted for weak discussion of number theory: no mention of Fundamental Theorem of Arithmetic, divisibility tests, odd and even numbers.

F. Rational and irrational numbers
   1. Terminology
   2. Decimal fractions
   3. Fractions in bases other than ten
   4. The language of percent
   5. Irrational numbers

Points deducted for lack of mention of:
- integers and operations in fractions (4 pts.)
- ratio and proportion (1 pt.)
- estimation, rounding (6 pts.)

V. Instructional Procedures

A. Brief introduction and summary lectures on the main topics
B. Diagrammatic and graphic demonstrations and explanations from the marker board
C. Daily assignments of problems to be completed for the next class session.
D. Class Participation.
E. Topic presentation

VI. Responsibilities of Students

A. Read textbook
B. Attend class
C. Timely preparation of assignments
D. Preparations for examinations
E. Group project and presentation-Make visual aid for some concept covered during the semester.
F. Cell phones are to be turned off when entering class and cannot be used as a calculator during quizzes or exam

VII. Evaluation

A. Two in-class exams are planned worth two-hundred (200) points each.
B. Three take-home exams are planned worth one-hundred (100) points each.
C. Quizzes-Three planned on days the Take Home Exams are due. (25 points each)
D. Attendance and class participation — 75 points (See attendance policy below.)
E. Final Exam — 300 points.
F. Please note: A grade of 'D' is not allowed in the course.
G. Grade determination: Divide the total points possible by the total points achieved to get a percentage.

Points	Percentage	Grade
1150-1035	90-100%	A
1034-920	80-89%	B
919-805	70-79%	C
Below 805	Below 70%	F

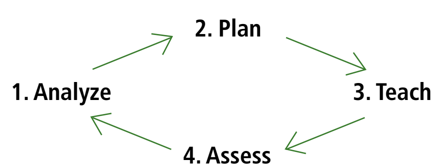
For more information on topics and subtopics in each of the four critical areas of mathematics, see Appendix B.

APPENDIX F: RUBRIC FOR EVALUATING MATHEMATICS METHODS COURSE TEXTBOOKS; TEXTBOOK EVALUATIONS

The focus of this study is the instruction provided to prospective elementary teachers in mathematics content courses. Our review process for textbooks for such courses was very systematic. Because our consideration of mathematics methods coursework was more circumscribed, our review process for textbooks was designed to provide only very basic information on most of the textbooks used in our sample, including their relative popularity among instructors.

There are important differences in the use of textbooks in mathematics content and methods courses. In content courses, generally only one text is required and it functions as a classic textbook. In mathematics methods courses, it is common to have several required texts and a few more recommended texts. The knowledge base for teaching mathematics generally or for any specific topic is quite thin, so there is no text that addresses a discrete methodology of mathematics instruction. In developing a composite text assignment, the instructor may consider how each textbook complements those that are used in a general methods course. Assigned texts may include references, case studies, personal narratives, and catalogs of resources to support classroom activities.

With the understanding that texts assigned in methods courses in our sample's programs may play different roles, we asked an experienced elementary mathematics teacher trainer to comment on the numbers and operations sections of textbooks from the perspective of a practitioner. The reviewer evaluated each one on the basis of how well it addressed the entire instructional cycle:



1. Analyzing data (quantitative and qualitative) and matching it to available information on best practices.
2. Planning coherent instruction by thinking through and scripting individual lessons. Taking into consideration long-term coherence, children's mathematical thinking, a variety of student performance levels, student motivation, *and the mathematical content*.
3. Teaching and mechanisms for the continuous improvement of teaching such as peer observation, microteaching, and videotaping.

4. Assessing student learning using student work and teacher observation and assessing one's teaching using collected data.

Those that addressed all of these aspects of teaching were categorized as "core" texts and are ranked in the chart below according to how well our evaluation suggests that they address this entire instructional cycle. Regardless of the manner in which a methods instructor wishes to blend various texts we believe that every prospective teacher needs to be introduced to at least one text addressing this cycle as the work of teaching, rather than simply being provided with texts that represent collections of resources on each particular element of teaching.

Other texts may address only one or two elements of this cycle and we address their value in doing so in the list of "supplementary texts." These books are listed by author alphabetically.

Core Textbooks

Textbook Title	Author(s)	**Comments**	Number of Courses in Which Textbook is Used
Teaching Mathematics to All Children	Tucker, Singleton, Weaver	Refreshingly readable, mathematically sound, coherent, and user friendly. Excellent exercises at the end of each chapter ask teachers to think through instructional decisions based on student needs. Models are strong and mathematically correct.	2
Teaching Problems and the Problems of Teaching	Lampert	Rich in description of the thought processes of both instructors and students in the problem-solving approach to teaching.	1
Learning Mathematics in Elementary and Middle Schools	Cathcart, Pothier, Vance, Bezuk	Not as coherent as one might wish, but contains much good information, especially on children's mathematical thinking.	3
Elementary and Middle School Mathematics: Teaching Developmentally	Van de Walle	An important book with much creativity, experience, and thought. Provides a developmental perspective on the cognitive development of children's mathematical thinking. More a resource for new ideas to support instruction than a main skeleton of methods of teaching. (See Appendix D for a mathematician's perspective on this text.)	30
Helping Children Learn Mathematics	Reys, Lindquist, Lambdin, Smith	While ideas are well-modeled, the transition to upper elementary math is not cohesive and a process for ongoing teacher improvement is lacking.	10
Mathematics Methods for Elementary and Middle School Teachers	Hatfield, Edwards, Bitter, Morrow	This book buries poorly developed mathematics under a sea of activities.	1
Guiding Children's Learning of Mathematics	Kennedy, Tips, Johnson	More of a reference than a textbook: a source of extra activities, brief insights, and explanations of educational terminology.	4

Supplementary Textbooks

Textbook Title	Author(s)	Comments	Number of Courses in Which Textbook is Used
About Teaching Mathematics	Burns	Important book in the history of math instruction reform with its encouragement of "meaning making," but not useful as a methods book.	2
Math: Facing an American Phobia	Burns	Presents a different vision of math instruction in nonthreatening, anecdotal prose.	2
So You Have to Teach Math	Burns	May be helpful to get teachers past their initial fear of teaching math.	1
Children's Mathematics	Carpenter, Fennema, Franke, Levi, Empson	Not suitable as a "stand-alone" text, but contains clear presentations of student thinking.	3
Classroom Discussions	Chapin, O'Connor, Anderson	Valuable in explaining techniques for structuring "math talk."	3
Teaching Number in the Classroom	Martland, Stafford, Stanger, Wright	A useful compendium of activities for developing early numeracy.	1
Beyond Arithmetic	Mokros, Russell, Economopoulos	Simply makes the case for a constructivist method of teaching.	1
Good Questions for Math Teaching	Sullivan, Lilburn	Contains questions of limited value as they are not contextualized within a mathematical framework.	2
Children Solving Problems	Thornton	Insightful book awakening teachers to the inner realm of young children's minds.	1
The Multicultural Mathematics Classroom	Zaslavsky	Helpful as a source of activities with which to enrich instruction.	1

APPENDIX G: THE DIFFERENCE BETWEEN MATHEMATICS COURSES INTENDED FOR A GENERAL AUDIENCE AND THOSE DESIGNED FOR TEACHERS

We assume that the nature of instruction in algebra in an elementary content mathematics course versus a general audience college algebra course is generally revealed by their textbooks.

Our purpose is to convey how elementary mathematics content courses and textbooks can ideally be designed to handle important topics with integrity and address mathematical structures even as they cover fewer topics and techniques than general audience courses. As our evaluations reveal, many programs and textbooks are deficient in their treatment of algebra. This problem, however, is not resolved by relegating coverage of algebra primarily or solely to a course that is not designed for teachers.

What can elementary content courses and textbooks do well? First, they can deal with the building blocks of the subject. Looking at *Mathematics for Elementary Teachers* by Beckmann, the textbook earning the highest marks for textbooks with a "stand-alone" algebra section of textbooks in our sample, we see four and one-half pages devoted to a discussion of "mathematical expressions, formulas, and equations." Only two of the thirteen algebra textbooks used in general audience courses in our library — which contains most of the textbooks used in general audience algebra courses found in institutions in our sample — discuss these basics at all. One devoted three pages to these topics (*Intermediate Algebra: Concepts and Applications*, Bittinger and Ellenbogen) and another, two and one-half pages (*Intermediate Algebra*, Tussy and Gustafson). Clearly the majority of writers of textbooks intended for use at the secondary or collegiate level do not think their readers need any reinforcement of basic concepts, yet these are precisely the concepts that an elementary teacher will using to frame pre-algebra instruction in the classroom.

Expressing quantities and their relationships through symbolic representation is the essence of algebra. Elementary teachers must be able to confidently convey to students both the capacity to use symbolic representation and an appreciation for its problem-solving power. Yet the near universal dread of "word problems" tells us that neither teachers nor students use or much appreciate this power. Beckmann devotes seven and one-half pages to a discussion of "solving equation with pictures and with algebra" with extensive instruction on how equations can symbolically represent relationships expressed in word problems. Both the Bittinger text and the Tussy text give simple glossaries to help translate words to symbols. These tables of handy mnemonics can not take the place of an actual understanding of the concepts and processes involved. Likewise, both texts devote three and one-half and five and one-half pages respectively to the topic of solving equations with a focus that is entirely procedural.

Elementary content textbooks can also deal with the nuances of even the simplest mathematical concepts in a way that both reinforces understanding and assists the prospective teacher to understand children's mathematical thinking. This sample from page 595 of the Beckmann text demonstrates this; no comparable discussion is found in the Bittinger or Tussy texts:

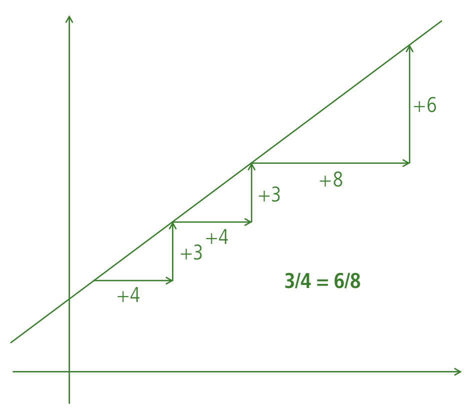
> *One source of difficulty in solving equations is understanding that the equals sign does not mean "calculate the answer." For example, when children are asked to fill in the box to make the equation*
>
> $5 + 3 = \square + 2$
>
> *true, many will fill in the number 8 because 5 + 3 = 8.*

Another excerpt from Beckmann demonstrates how elementary content textbooks can convey a deep understanding with a clarity that allows prospective teachers to perceive and retain the core concepts that will frame their own instruction. The one and one-half page section (pages 640-641) from Beckmann below precedes her statement that every linear function has a formula of the form $f(x) = mx + b$.

> *Consider a linear function. By definition, its graph is a line. We can use this line to form many different right triangles that have a horizontal and a vertical side, as shown in Figure 13.45. Because the horizontal lines are all parallel, the angles they form with the graph of the function are all the same. Since they are right triangles, all three angles of each of these triangles must be equal, and so all such triangles are similar. (See Section 9.4.) Therefore, the ratios of the lengths of the horizontal sides to the lengths of the vertical sides are equal for all these triangles. The lengths of horizontal sides of these triangles represent "changes in inputs" because the horizontal axis represents inputs; the lengths of vertical axis represents outputs. Therefore, for a linear function, the ratio of the change in input to the corresponding change in output is always the same.*

Figure 13.45

FOR LINEAR FUNCTIONS THE RATIO OF THE CHANGE IN INPUT TO THE CORRESPONDING CHANGE IN OUTPUT IS ALWAYS THE SAME DUE TO SIMILAR TRIANGLES



> *We have seen that for linear functions, the ratio of the change in input to the change in output is always the same. We will now use this fact to deduce that every linear function has a formula of a certain type. Let* b *be the location on the* y*-axis where the graph of the function crosses the* y*-axis. Suppose that when the input is increased by 1, the output increases by* m*. Because the ratio of the change in input to the corresponding change in output is always the same, if the input increases by* x*, the output increases by* mx*. Therefore, when the input is* x*, the output is*
>
> $$mx + b$$
>
> *as indicated in Figure 13.46. So every linear function has a formula of the form*
>
> $$f(x) = mx + b$$
>
> *for some numbers* m *and* b*. The number* m *is called the slope of the line; the number* b *is called the* y*-intercept of the line.* *slope* y*-intercept*

This use of similar triangles to graphically demonstrate the meaning of a constant slope, which naturally leads into the depiction of a line as pairs of coordinates (x, $mx + b$), explains the *why* of the equation of a straight line in a coherent, consolidated way that is readily accessible to prospective teachers. From this, the *how* readily follows.

In contrast, Bittinger and Tussy devote three pages and two pages respectively to this same exposition. Bittinger gives a very complete discussion of the *how*, but the *why* is conspicuously absent. The discussion in Tussy is similar to Bittinger's, but even more formulaic, as different forms for the equation of a line (point-slope and slope-intercept) are presented with a minimum of justification. The fundamental concept of a linear function is almost camouflaged by the conceptual and computational density of its surroundings, and that is true even before we consider how these linear functions sections are themselves dwarfed by the texts' coverage of polynomial, rational, radical, exponential and logarithmic functions, all less essential for the elementary teacher's preparation for the classroom.

APPENDIX H: SAMPLE PRACTICE TEACHING ASSIGNMENTS FROM SYLLABI FOR MATHEMATICS METHODS COURSES

good articulation of course objectives

A PRACTICE TEACHING ASSIGNMENT THAT PUTS CONVEYING MATHEMATIC CONTENT 'FRONT AND CENTER'

OVERVIEW AND PURPOSE OF THE LESSON-BASED ASSESSMENTS

In mathematics methods this semester, You have had multiple opportunities to observe and analyze mathematics teaching, as well as do the work of teaching in ways that have allowed you to learn from and improve you practice. Your coursework has focused on important domains of teaching practice-such as leading discussions, representing ideas, planning instruction, and assessing students-and has been designed to separate the work of mathematics teaching to make it more manageable for learning and practicing. A teacher's day-to-day practice, however, is much more integrated. Becoming a skillful teacher entails bringing the domains of practice together into integrated acts of instruction that attend to mathematics and student learning in principled ways.

The culminating performance assessments of your mathematics teaching skills have been organized within a typical structure, a mathematics lesson. Teaching a lesson brings together thoughtful planning, skillful orchestration of discussion, and insightful analysis of assessment information-the very domains in which you have been developing skill throughout the semester. The three lesson-based culminating performance assessments draw heavily upon the skills that you have been developing in these areas. They also provide an opportunity to show your use of important principles to guide you mathematics teaching. The course content matrix below illustrates the location of the lesson-based performance assessments in relation to the course domains and principles.

Professor puts content first

Guiding Principles

Domains of Teaching **Practice**	Attending to the **integrity of the mathematics**	Committing to the learning and achievement of **all students**	Establishing and managing a **productive learning environment**	Learning from and systematically **improving practice**
Planning mathematics lessons	Lesson Analysis Conference			
Leading a whole class discussion about mathematics	Leading a Discussion in a Mathematics Lesson			
Assessing students knowledge, skill, and disposition	Assessing Students Through an End-of-Class Check			
Representing mathematical ideas				

Instills critical need to formative assessments

The following pages describe the steps required to complete the three lesson-based culminating performance assessments for the mathematics methods course:

- Lesson analysis conference in which you will carefully analyze the mathematics lesson that you will be teaching and discuss your lesson analysis and modifications with your course instructor.
- Leading a discussion in a mathematics lesson in which you will lead a whole class discussion of a mathematical concept, procedure, or problem from you lesson.
- Assessing students through an end-of-class check in which you will use an assessment prompt to assess what students learned about the mathematical content of your lesson.

DIRECTIONS FOR COMPLETING THE THREE LESSON-BASED CULMINATING PERFORMANCE ASSESSMENTS

Make arrangements

organized

1. Make arrangements with your cooperating teacher to teach a complete mathematics lesson (between November 20 and the end of the term). Send an email with the date you selected as soon as possible
2. Once you know the date of your lesson, schedule a 20-minute meeting with you math methods instructor to discuss your analysis of the lesson. The date you choose should be at least one day before you will teach the lesson.

   encourages feedback, retention

3. Make arrangements to video (or audio) record the discussion portion of you lesson. If you need help locating equipment, check with your cooperating teacher or field instructor to see if he or she can help you borrow equipment from your school or district. In addition, it is often easier to have someone else video tape while you are teaching, so you might check if your cooperating teacher, field instructor, colleague or methods instructor is available to help you with this. While video will provide you with the richest documentation of you teaching, you may make an audio record if you are unable to locate video equipment.

Plan your lesson

Emphasis on the Mathematics

4. Carefully analyze the textbook lesson that you will be teaching. Explore the curriculum materials to gain a sense of how the lesson builds upon prior lessons and sets the stage for lessons that follow it. Then analyze the lesson itself, attending to the elements in our textbook lesson analysis table. Determine the mathematical content and goals of the lesson and think about students as learners of this content. Then consider the specific content and features of the lesson such as context, examples, language, task progression, and/or representations and tools in light of the integrity of the mathematics and concern for the learning of all students. **Document your insights on the textbook lesson analysis table**. If your analysis indicates that you need to modify the lesson, document you modifications on a copy of the lesson, on a separate sheet, or in a draft of the lesson plan you will use.
5. Plan the enactment of you lesson, and gather or make any materials you will need. Your lesson must include a **whole class discussion** of a mathematical concept, procedure, or problem. In addition, you should design an **end-of-class check**. Your prompt should be a question that assesses student' understandings of or skills with the specific mathematics content of the lesson. You may want to use of modify a prompt from you curriculum materials or other resources. Be sure to consider the purpose for you prompt, how long it will take students to answer it (aim for less than 5 minutes), and how you plan to pose the prompt to your class.

Priority assigned to student understanding of specifc content

6. Submit the following items **at least 48 hours** before your meeting:
   - Completed textbook lesson analysis table
   - Copy of the textbook lesson with a record of any modifications you decided to make
   - Draft of your en-of-class check prompt

Lesson Analysis Conference Culminating Performance Assessment

7. Meet with your methods instructor to discuss the analysis of your lesson. Come at you scheduled time and be prepared to:
   - Share a brief overview of your classroom setting (including details like grade level and how your lesson fits with what students have been and will be learning).
   - Discuss your analysis of the mathematics of the lesson and students as learners of this content Explain how your en-of=class check relates to the mathematical and instructional purposes of your lesson.
   - Share your principled consideration of specific lesson content and features (i.e., page 2 of the textbook lesson analysis table). In addition, be prepared to discuss:
     - What modifications you decided to make or why you decided not to alter the lesson
     - Why you attended to particular features of the lesson and not others
8. Use insights form your conference to revise your lesson plan

Feedback from Methods instructor

Reflecting on Mathematics and learning mathematics

Teach and document your lesson

9. Teach the lesson you planned. Be sure to conduct a discussion and use an end-of-class check at the conclusion of your lesson.

10. Video (or audio) record the discussion segment. You are encouraged to record the entire lesson if it is convenient.

11. Collect or make copies of you students' responses to your end-of-class check, as well as any other student work produced during the lesson.

Leading a Discussion in a mathematics Lesson Culminating Performance Assessment

12. Play your entire recording, focusing on the segments involving whole class discussion. Make note of the times that your discussion begins and ends and, if applicable, cue your tape to the launch of your discussion. Your discussion will be evaluated for it inclusion of the following elements:
    - Does the mathematics problem allow for discussion?
    - Does the teacher launch the discussion to elicit initial contributions?
    - Is the discussion focused on mathematics?
    - Does the teacher solicit broad participation?
    - Does the teacher use a variety of moves that:
        - Probe students' contributions
        - Connect students' ideas
        - Encourage students to consider and respond to classmates' ideas
        - Guide students' to reason mathematically
        - Extend students' thinking
    - Does the teacher conclude the discussion?

    Note: You do not need to write anything for (#12). Your methods instructor will look for these elements when viewing (or listening to) your discussion.

Mathematics is central focus of analysis of interaction

13. In addition, choose <u>one</u> aspect of leading a whole class discussion form the list below that you feel you were able to skillfully perform in you discussion:
    - Purposefully using questions to elicit, probe, and connect students' mathematical ideas
    - Supporting students to consider and respond to their classmates' mathematical ideas
    - Helping students make explicit connections between representations and/or solution strategies
    - Concluding the discussion in a way that highlights the main mathematical content of the lesson and goals of the discussion
    - Fielding a student response that you anticipated or did not anticipate in your planning
    - Attending to and engaging all students' participation in the discussion
    - Mediating the context of the lesson to attend to issues of equity
    - Improving the practice you identified as something you wanted work on from you mini-problem discussion

Again mathematics is central focus of analysis of interaction

14. White a one-to two-paragraph explanation of how the discussion reflects your ability to skillfully perform the aspect of leading a while class discussion that you selected in #13. Use <u>specific</u> examples from the discussion (with references to times) and the course teaching principles, readings, or other course work to identify aspects of skilled performance and to support your explanation. Please include your name and grade level on your write-up.

Reflection on content of lesson

15. Turn in the components of the *Leading a Discussion In a Mathematics Lesson* Culminating performance assessment by **Friday, December 15, at 5 pm**:
    - Video or audio recording of discussion, cued to the discussion segment
    - Analysis of one aspect of your discussion (#14)
    - Other relevant materials: copies of student work, lesson plan, etc.

Assessing Students Through an End-of Class Check culminating Performance Assessment

16. Review students' responses to your end-of-class check prompt, and analyze what you learned about individual students, as well as the class as a whole, from their responses.
17. Make a record of class performance on the end-of-class check. For example, you could make lists of students who fall into different performance categories; who used particular methods; who made particular errors; who seem to have mastered a skill. Perhaps a simple frequency table will suffice.

Analysis of assessment of student learning

18. Write a reflection about your assessment of students' understandings based on your end-of-class check. Please include your name and grade level on your write-up:
    - Describe your end-of-class check: What prompt did you use and what was your purpose?
    - Interpreting and assessing student thinking: List three insights that you learned about your students from your end-of-class check. At least one of your insights should be about an individual student, and at least on should be about the class as a whole. For each of you three insights:
        - Provide specific examples or evidence from your performance record and/or individual responses to support your assessment.
        - Explain why knowing this information is useful to you as a teacher.

19. Turn in the components of the *Assessing Students' Understandings* culminating performance assessment by **Friday, December 15, at 5 pm**:
    - Class performance record (#17)
    - Written reflection (#18)
    - Copies of student responses to your end-of-class check

A PRACTICE TEACHING ASSIGNMENT THAT DOES NOT EVEN MENTION MATHEMATICS

The problem is not length but that there's no evidence that the content matters

SELF-EVALUATIONS OF CLASSROOM TEACHING

The intent of this project is not to have you demonstrate "flawless" teaching. To the contrary, the intent is to have you accurately assess and modify your teaching.

Two assignments fall under this heading; "classroom environment" and "teacher behaviors". The "classroom environment analysis" focuses on how you, via your interaction behaviors, foster a positive classroom culture. You must submit a video clip that clearly captures your interactions with <u>entire class</u> and/or with students in <u>small groups</u>. The "teacher behaviors analysis" focuses on your questioning and response patterns. For this analysis, you must submit a videotape of yourself working with the <u>entire class</u> and/or with students in <u>small groups</u>. For both analyses, you must select a continuous 15-minute section of your taped teaching and complete a quantitative and qualitative assessment.

Where's the math ????

Once both evaluations are completed and turned in, schedule a meeting with me, so we can view your clip together. Be prepared to discuss your teaching in-depth during this meeting. Please know the "new" course schedule provides time for meetings to be scheduled before or after school, or after class time here at CC (If you have to go to your school placement late or leave early because of this meeting, be sure to advise your cooperating teaching before hand.) During this meeting, you may be asked questions concerning your teaching, in which responses are added to the qualitative part of one or both evaluations.

* If you have completed these analyses in another class of mine, the qualitative part of both evaluations should focus on how your teaching has progressed from the first set of evaluations to the second, with special attention paid to future areas of growth.

Feedback from methods instructor is given

APPENDIX I: ADDITIONAL QUESTIONS SHOWING THE DIFFERENCE BETWEEN MATHEMATICS PROBLEMS FOR CHILDREN AND THOSE FOR TEACHERS.

CONTRASTING PROBLEMS: The mathematics that teachers need to know – and children do not

Mathematics questions CHILDREN should be able to answer – taken from actual college course assessments.

1a. Which of the following statements is true?
- a. Every decimal number can be expressed as a fractional number.
- b. Every fractional number can be expressed as a decimal number.
- c. Neither (a) nor (b) is true.
- d. Both (a) and (b) are true.

2a. Which of the following could be used to determine if a number is divisible by 36?
- a. It is even and divisible by 18.
- b. It is divisible by both 4 and 9.
- c. It is divisible by both 3 and 12.

Justify your choice.

3a. The prime factorization of a number is $17^4 \cdot 11^2 \cdot 2$
- a. Name one prime factor of the number.
- b. Name two composite factors of the number.
- c. How many total factors does the number have?
- d. Is this number even or odd? How do you know?

Mathematics questions that are closer to hitting the mark for what TEACHERS should be able to answer – taken from actual college course assessments.

1b. True or false?
Let *a*, *b*, *c*, and *d* be whole numbers. If $d = 0$ and $a = 0$, then the number below is a repeating decimal:

$$\frac{3\Pi^d(\sqrt{4})^a}{(2\sqrt{2})^a 5^b 7^c}$$

2b. What is the least number divisible by each natural number less than or equal to 20?
- a. 23 · 33 · 52 · 72 · 11 · 13 · 17 · 19
- b. 24 · 32 · 5 · 7 · 11 · 13 · 17 · 19
- c. 24 · 33 · 5 · 7 · 11 · 13 · 17 · 19
- d. 23 · 32 · 5 · 7 · 11 · 13 · 17 · 19

3b. Prove that the product of two odd numbers is odd. Your proof should begin with a clear definition of "odd," and then be stated in simple, complete sentences, showing how the result follows logically from known arithmetic facts.

4a. Give a property each illustrates:
a. $(x + y) + 3 = 3 + (x + y)$
b. $(x + y)3 = 3x + 3y$

5a. What is the area in square centimeters of the right triangle with hypotenuse of 26 cm, and legs of 10cm and 24 cm?
a. 60   b.120

6a. State whether this is always true, sometimes true, or never true: A rectangle is a square.

7a. Before the mean can be computed, we need to determine which of the following?
a. frequency
b. variation
c. mode
d. median

4b. Let @ and # be the binary operations defined by

a@b = 2a + b and a#b = a + 2b,

respectively. Give an example to show that the left distributive law of the operation # over the operation @,

a # (b @ c) = (a # b) @ (a # c),

does not work.[1]

5b. A regular hexagon has sides measuring 3.5 cm. Find the area of the hexagon. Can one of the special triangle formulas be used to find the area? If so, which one and why? If not, why not?

6b. Explain why a trapezoid which has a pair of opposite congruent angles must be a parallelogram.

7b. Three students were conducting a survey to determine how many songs UM students had downloaded from Internet sources in the past six months, along with the sources of the downloads. The students findings are in the frequency distribution table below. Copy the table into your answer booklet then fill in the correct relative frequencies and cumulative frequencies.

No. of downloads	Frequency	Relative frequency	Cumulative frequency
0 - 100	33		
101 – 200	96		
201 - 300	87		
301 - 400	63		
401 - 500	21		

1 Modified slightly for clarification.

EXIT WITH EXPERTISE:
Do Ed Schools Prepare Elementary Teachers to Pass This Test?

The problems that follow are by no means exhaustive, but suggest the broad goals of the mathematics content coursework in an elementary teacher preparation program. In their current form, the problems may not be suitable for use in a standardized examination with time limitations. Nonetheless, every prospective and practicing elementary teacher should be able to solve them without use of a calculator.

Answer key on page 5.

1. Let n and m be odd numbers.
- **a.** Give a picture proof (e.g., array model) that the product nm is odd.
- **b.** Prove algebraically that n^2 is odd.
- **c.** Prove algebraically that when n^2 is divided by 4, the remainder is 1.
- **d.** Prove algebraically that when n^2 is divided by 8, the remainder is 1.
- **e.** Find an odd n such that n^2 divided by 16 leaves a remainder that is not 1.

2. Make up two word problems for which division is required, both involving cookies:
- **a.** Express the partitive meaning of division for 50/10.
- **b.** Express the measurement meaning of division for 50/10.

3. **a.** Find a rational number between 3.1 and 3.11.
- **b.** Find an irrational number between 3.1 and 3.11.

4. *This problem investigates the interaction of the two operations of adding and rounding. It deals with two-digit numbers and the operation of rounding them to the nearest 10. The point is to compare the sum of the two rounded numbers with the rounding of their sum. Let the rounding rule be that 5 rounds up (i.e., 45 would round to 50).*

Think about adding a two-digit number with tens digit 3 to a two-digit number with tens digit 4. Suppose the ones digit of each can be any digit at random (i.e., with equal likelihood).

- **a.** What is the probability that the sum will round to 80?
- **b.** What is the probability that the sum will round to 70? to 90?
- **c.** Suppose one of the numbers rounds to 30, and one rounds to 40. What is the probability that the sum rounds to 70? What is the probability that the sum rounds to 80?
- **d.** Suppose that one number rounds to 40 and one to 50. What is the probability that the sum rounds to 90? to 80?
- **e.** Suppose both numbers round to 40. What is the probability that the sum rounds to 80?

5. The letters a and b are digits (i.e., 0, 1,…,9). Find all choices of a and b so that $2a1181b4$ is divisible by 12, but not divisible by 9.

6. For each whole number n find the greatest common divisor (GCD) of $3n+1$ and $n+1$.

7. Explain, as you would to a 4th grade class, each step in the multiplication of 32 by 14.

8. **a.** Give a realistic context where signed numbers are relevant and that justifies $(-a)(-b) = ab$.
- **b.** Based on the definition of the additive inverse ($-a$ is the number such that $a + (-a) = 0$) and the distributive rule ($a* (b+c) = ab + ac$ for all numbers a, b, and c), give a formal argument to justify $(-a)(-b) = ab$.

9. Make up first grade word problems of the following types:
- **a.** The take-away interpretation for finding 15 – 7.
- **b.** The part-whole interpretation for 26 – 4.
- **c.** The comparison interpretation for 17 – 5.

10. Write all the three digit numbers that have one digit equal to 1, one digit equal to 2, and one digit equal to 3. Add them all up.

a. Show that the sum is divisible by 37.

b. What would have happened if you used different digits? Try the same problem with 2, 7 and 8.

c. Will the sum always be divisible by 37?

11. The population density of a region is the total population of the region divided by the area of the region (usually expressed in square miles or square kilometers). Asia (excluding Siberia) contains about 20% of the world's land area, but is home to about 60% of the world's people.

a. What is the ratio of the population density in Asia to the population density of the rest of the world?

b. If the overall population density of the world is 110 people per square mile, what is the population density of Asia? What is the population density of the rest of the world?

12. Write the repeating decimal $0.572\overline{72}$ as a fraction.

13. A store has a sale with a *d*% discount and must add a *t*% sales tax on any item purchased.
Which would be cheaper for any purchase:

a. Get the discount first and pay the tax on the reduced amount.

b. Figure the tax on the full price and get the discount on that amount.

Justify your answer.

14. You determine that 2/3 of a gallon of blue paint and 1/2 of a gallon of red paint will make a pleasant shade of purple. In a manner that can be understood by 5th graders, solve for the number of gallons of blue paint and red paint will you need to make 84 gallons of this purple paint.

15. John's shop sells bicycles and tricycles. One day there are a total of 176 wheels and 152 pedals in the shop. How many bicycles are available for sale in John's shop that day? Solve arithmetically and algebraically.

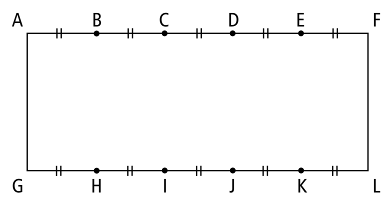
16. It takes one corn mill 6 minutes to grind a 50 pound bag of corn into cornmeal, while it takes a slower mill 9 minutes to grind the same bag of corn. If both mills are working at the same time, how long would it take to grind 1500 pounds of corn?

17. Make up a short word problem that builds the given expression in the given context. Be sure to make clear what the *x* represents: $(240 - x)/50$ as the time needed to complete a trip to another city.

18. Which is larger: 10^{10} or 3^{20}? Justify your answer using comparisons of expressions that contain identical bases or identical exponents.

19. The day you begin to store a watermelon in your basement it weighs 10 pounds and is 99% water. You forget about it for a while and when you bring it up from the basement, it is 98% water. What does it weigh?

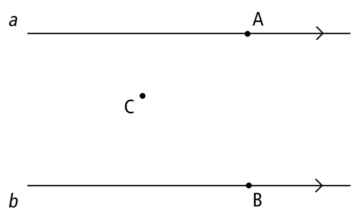
20.



Let b *represent the base of the rectangle and* h *r epresent its height.*

A different polygon is drawn within each of three rectangles with vertices AFLG.
Polygon No. 1: A parallelogram with vertices DFIG
Polygon No. 2: A trapezoid with vertices EFJG
Polygon No. 3: A triangle with vertices ALH
How do the areas of the three polygons compare? Justify your answer.

21. Lines *a* and *b* are parallel. Connect points A and C, and points B and C with line segments. The measurement of the acute angle with its vertex at point B created by CB is 40º; the measurement of the acute angle created by CA with its vertex at point A is 30º. Find the measurement of ∠ACB.



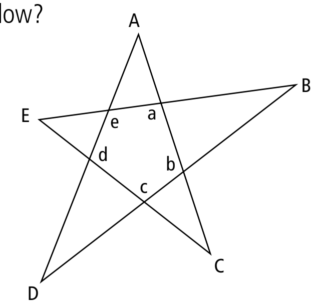
22. You are visiting a garden that begins at point A and ends at point D, with two posts (points B and C) between points A and D. There are two ways to walk through the garden: 1) a path that starts at point A and ends at point D, traveling along the circumference of a circle that has segment AD as a diameter, and 2) a path that starts at point A and ends at point D, traveling along half of the circumferences of each of three circles, the first with diameter $\overline{DC} = c$, the second with diameter $\overline{CB} = b$, and the last with diameter $\overline{BA} = a$.

You walk the top path going from point A to point D and the other path along the three semicircles going from point D back to point A. Which of these two paths is longer? Explain why.

23. Which leaves more open space in relationship to the total area of the hole?

- A square peg in a round hole, where the diameter of the round hole equals the length of the diagonal of the square peg.
- A round peg in a square hole, where the length of a side of the square hole equals the diameter of the round peg.

24. What is the sum of the measures of the angles at the vertices A, B, C, D, and E of the five-pointed star below?



25. Suppose you are looking down a road and you see a person ahead of you. You hold out your arm and sight the person with your thumb, finding that the person appears to be as tall as you thumb is long. Let's say that your thumb is 2 inches long, and that the distance from your sighting eye to your thumb is 22 inches. If the person is 6 feet tall, then how far away from your sighting eye is the person (distance expressed in feet).

26. a. Describe the symmetries of the square. How many are reflections in lines? How many are rotations?

b. One kind of symmetry transformation of the square is a reflection in the perpendicular bisector of two opposite sides. Another kind is reflection in the line joining two opposite vertices. Describe the symmetry transformation that results from doing a transformation of the first type, followed by a transformation of the second type.

27. Draw a parallelogram that has a base length of 1 unit, an area of 1 unit squared, and a perimeter of 36/5 units

28. From a point O that is located between points A and B on the line AB, draw the ray OC, where C is a point not on line AB. Let the ray OM bisect the angle AOC and let the ray ON bisect the angle BOC. What is the measure of the angle MON? Justify your answer.

29. A cylindrical container can hold 2 quarts of water. If its radius and height are both doubled, how much water can it hold?

30. When rolling two dice is it likelier to have the two faces add to 6 or add to 7?

Answer Key

EXIT WITH EXPERTISE: Do Ed Schools Prepare Elementary Teachers to Pass This Test?

1. If n is an odd number, it can be represented as $2w+1$, where w represents a whole number (0,1,2…).

a. As a generic example consider:

○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○

There are an odd number of rows, so pair them going down to get an even number with just the last row left over. This has an odd number of circles so pair them from the left, leaving the one in the lower right hand spot unpaired, showing that the remainder when dividing by 2 is 1 and the product is odd.

b. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 2(2w^2+2w) + 1$ so n^2 is odd. Helpful reminder for (b) and (c): In division with a remainder, when dividing by a number k, the result is a whole number and a remainder, with the remainder less than k (and greater than or equal to 0).

c. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 4(w^2+w) + 1$ Since w^2+w is a whole number and 1 is less than 4, the remainder when dividing by 4 is 1.

d. $n^2 = (2w+1)^2 = 4w^2+4w+1 = 8[(w^2+w)/2] + 1$. The expression $w^2+w = w(w+1)$, and either w or $w+1$ is even, so $(w^2+w)/2$ is a whole number. Thus the remainder when dividing by 8 is 1.

e. Many odd numbers when their square is divided by 16 leave a remainder that is not 1. The number 3 is the least odd number that satisfies this condition: $3^2 = 9$, and when this is divided by 16 the remain der is 9.

2. **a.** Ten bags contain a total of 50 cookies. How many cookies are in each bag?

b. You need to buy 50 cookies for a party. The cookies come in packages of 10. How many packages do you need to buy?

3. **a.** Because 3.1 = 3100/1000 and 3.11 = 3110/1000, 3.105 (3105/1000) is one possible rational number between the two values.

b. Every non-negative rational number when expanded as an infinite decimal is ultimately repeating. The number 3.1010010001…, with one more zero between the "1" at each stage in its expansion must be irrational. Alternatively, $\sqrt{2}$ (an irrational number) can be used to construct an irrational number between 3.1 and 3.11: Since $\sqrt{2} < 2$ and $3.11 - 3.1 = .01$, if we add $.01*(\sqrt{2}/2)$ to 3.1, the sum will be less than 3.11. So $3.1 + .005\sqrt{2}$ is an irrational number between 3.1 and 3.11.

4. The sums which will round to 80 can be written in a systematic way allowing all of the questions to be answered by inspection. A portion of the full table follows:

49	30	31	32	33	34	35				
48	30	31	32	33	34	35	36			
47	30	31	32	33	34	35	36	37		
46	30	31	32	33	34	35	36	37	38	
45	30	31	32	33	34	35	36	37	38	39
44		31	32	33	34	35	36	37	38	39
43			32	33	34	35	36	37	38	39
42				33	34	35	36	37	38	39
41					34	35	36	37	38	39
40						35	36	37	38	39

a. P = 3/4. There would be 100 items if all were written, and there are 10 missing in the upper right hand corner and 15 missing in the lower left hand corner, so there are 75 out of 100 which round to 80 after being added. Thus, the probability is 75/100 = 3/4.

b. P (sum rounds to 70) = 15/100 = 3/20
P (sum rounds to 90) = 10/100 = 1/10
There are 15 out of 100 which round to 70 after being added, and 10 out of 100 which round to 90 after being added.

c. P (sum rounds to 70) = 15/25 = 3/5
P (sum rounds to 80) = 10/25 = 2/5
If both numbers round down, they are in the lower left hand quarter of the table, which has 25 items. The 15 missing ones round to 70 when added and the remaining 10 round to 80 when added, so 15 out of 25 and 10 out of 25 are the chances.

d. P (sum rounds to 90) = 10/25 = 2/5
P(sum rounds to 80) = 15/25 = 3/5
If both numbers round up, they lie in the upper right hand quarter, which is treated the same way: 10 out of 25 round to 90 when added and 15 out of 25 round to 80 when added.

e. P(sum rounds to 80) = 1
The lower right hand quarter contains the numbers in question, and all of them round to 80 when added.

5. The possible values for *a* and *b* must yield a number that is divisible by 3 and by 4, but that does not include a number that is also divisible by 9. The possible values for *b* are 0, 2, 4, 6, and 8 (since all result in a number whose last two digits are a number divisible by 4). Using the fact that the sum of the digits must be divisible by three for the number to be divisible by 3, but the digits must not be a sum divisible by 9 to avoid a number divisible by 9, the values can be paired with *a* values as follows: $b = 0$, $a = 4$ or 7; $b = 2$, $a = 2$ or 5; $b = 4$, $a = 0$, 3, or 9; $b = 6$, $a = 1$ or 7; $b = 8$, $a = 5$ or 8.

6. If the number f divides $n+1$ and $3n+1$, then it also divides $3(n+1) = 3n+3$ and $3n+1$. So f also divides $(3n+3) - (3n+1) = 2$. Therefore, $f = 1$ or 2. If n is even, then $n+1$ is odd, and is not divisible by 2. Hence, when n is even, the GCD is 1. If n is odd then $3n$ is also odd, so $n+1$ and $3n+1$ are both even. In this case, 2 divides both $n+1$ and $3n+1$, so it is the GCD.

7. Multiplying 32 by 14 is the same as multiplying 32 (4 + 10).

The first product line is found by multiplying 32 by 4. This process can be broken down as well as it is the same as multiplying 4 (2 + 30). We multiply 2 ones by 4 ones and get a product of 8. We then multiply 30 by 4 and get a product of 120. The sum of 8 and 120 is 128.

The second product line is found by multiplying 32 by 10. The process can be broken down; it is the same as multiplying 10 (2 + 30). We multiply 10 by 2 for a product of 20 and 10 by 30 for a product of 300. The sum of 20 and 300 is 320.

The sum of the two products of 128 and 320 is 448.

3 2	
x 1 4	
8	4 x 2
1 2 0	4 x 30
2 0	10 x 2
3 0 0	10 x 30
4 4 8	

3 2	
x 1 4	
1 2 8	4 x 32
3 2 0	10 x 32
4 4 8	

8. a. Many answers are possible. Here is an example from a game playing situation. You are playing a board game and hold two cards that would each penalize you by making you move your marker back three spaces. Due to another card you pick, both of the penalty cards are put back in a deck and you need not move back at all! Losing two cards that each represented a penalty of three spaces puts you six spaces ahead of where you would have been had they been used: (-2)(-3) = 6.

b. $(-a)(-b) + (-a)(b) = (-a)(-b + b)$ by the distributive rule, but $-b + b = 0$ so $(-a)(-b)$ is the additive inverse to $(-a)(b)$. On the other hand $(a)(b) + (-a)(b) = (a + -a)(b) = (0)b = 0$ by the distributive rule, so $(a)(b)$ is the additive inverse to $(-a)(b)$. Since an integer has only one additive inverse, it follows that $(a)(b) = (-a)(-b)$.

9. a. How many of 15 socks are left in a laundry basket if seven are removed?

b. Four children in a class return to the classroom. If there are 26 students in the class, how many more need to return to the classroom before everyone is present?

c. Seventeen dogs stayed in a pet hospital Monday night. Five dogs stayed Tuesday night. How many more dogs stayed at the hospital Monday night?

10. The three digit numbers are: 123, 132, 213, 231, 312, 321. The sum is 1332.

a. 1332/37 = 36

b. The sum is 3774. 3774/37 = 102

c. Yes. Each of the three columns of digits in the three-digit numbers will add up to the same number, n. The sum will be $n(100 + 10 + 1)$ or $n(111)$. Since 111 is divisible by 37, $111n$ is divisible by 37.

11. a. The ratio is 6:1. If Asia has 3/5th of the world's population and 1/5th of the world's area, and the rest of the world has 2/5th of the world's population and 4/5th of its area, then the ratio of population density of the two is 3: 1/2 or 6:1.

b. The population density of Asia is 330 people per square mile. The rest of the world has a population density of 55 people per square mile. The ratio of Asia's population density to the world's overall density is 3:1, so Asia's density must be 330 people per square mile. The ratio of Asia's population density to the rest of the world's is 6:1, so the rest of the world must have a population density of 55 people per square mile.

12. The fraction is 567/990 (= 63/110). Subtracting the decimal 0.572 (m) from 100 times the decimal ($100m$), produces the equation $99m = 56.7$. Then $m = 56.7/99$ or 572/990.

13. Neither is cheaper since both approaches yield the same total purchase price. To determine this, let p represent any purchase price:

a. Discounted price: $p - p*(d/100) = p(1-d/100)$
Tax on discounted price: $p(1-d/100)\ (t/100)$
Adding the two and simplifying: $p(1-d/100) + p(1-d/100)(t/100) = p(1-d/100)(1 + t/100)$

b. Full price with tax: $p + p*(t/100) = p(1+t/100)$
Discount on full price with tax:
$[p+p*(t/100)]*d/100 = p(1+t/100)(d/100)$
Subtracting the discount from the full price and simplifying: $p(1+t/100) - p(1+t/100)(d/100) = p(1+t/100)(1-d/100)$

These are the same since $a*b = b*a$

14. Since 2/3 = 4/6 and 1/2 = 3/6, for every 4 gallons of blue paint you need 3 gallons of red paint to make the right purple mixture. Putting these amounts of paint together will generate 7 gallons of purple paint. Mixing 12 of these 7-gallon batches of paint will make the 84 gallons desired, so you will need 12*4 = 48 gallons of blue paint and 12*3 = 36 gallons of red paint.

15. There are 52 bicycles in the shop.

Solved arithmetically:
Each bicycle has two wheels and each tricycle has three wheels, and both have two pedals. For each tricycle, there is one more wheel than pedals. There are 176−152 = 24 extra wheels, so there are 24 tricycles. These have 24*3 = 72 wheels, so the number of wheels on bicycles is 176−72 = 104. The number of bicycles is half the number of wheels, 104/2 = 52.

Solved algebraically:
Let b represent the number of bicycles in the store and t the number of tricycles. Equation A, developed using number of wheels: $2b+3t = 176$. Equation B, developed using number of pedals: $2b+2t = 152$. Subtracting equation B from A: $1t = 24$. Substituting this value for t into equation B and solving for b, $b = 52$

16. It would take 108 minutes to grind the 1500 pounds of corn. The combined rate of the two mills is (150 lb./18 min.) + (100 lb./18 min.) = 250 lb./18 min. Since 1500 = 6 times 250, it will take six times as long, or 6*18 = 108 minutes to grind 1500 pounds.

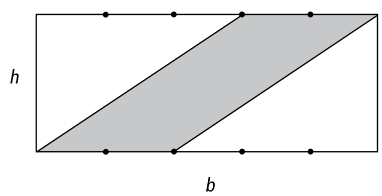
17. You are driving at 50 miles per hour to a city 240 miles away in two periods of driving. In the first period, you travel x miles. How many hours will it take to complete the trip?

18. 10^{10} is larger; $3^{20} = (3^2)^{10} = 9^{10}$; $9^{10} < 10^{10}$

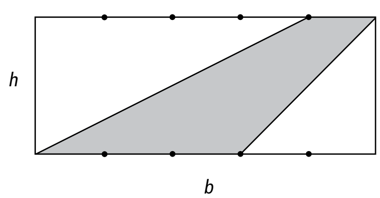
19. The watermelon weighs five pounds. Initially there were 9.9 pounds of water and .1 pound of solids. After a while there will be less water, but still .1 pound of solids, which represent 2% of the watermelon's total weight: .1 = .02*x*, where *x* represents the watermelon's weight: *x* = 5.

20. All the polygons have the same area: $A_1 = A_2 = A_3$

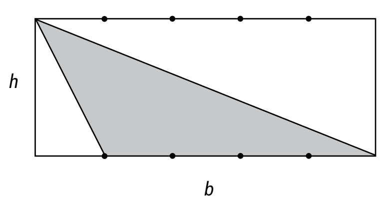
Area of parallelogram: $A_1 = 2/5b*h$



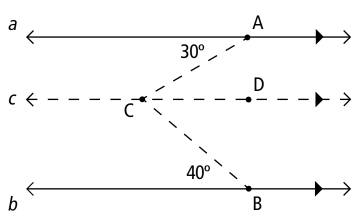
Area of trapezoid: $A_2 = 1/2h\,(3/5b + 1/5b) = 1/2h * 4/5b = 2/5b*h$



Area of triangle: $A_3 = 1/2\,(4/5b)*h = 2/5b*h$



21.



Angle ACB measures 70º.

Different approaches are possible, but one approach is to draw an auxiliary line[1] parallel to lines *a* and *b* through point C and add point D to line *c*:

$m\angle ACD = 30^\circ$ (This is an alternate interior angle to the acute angle with vertex A on line *a*.)

$m\angle DCB = 40^\circ$ (This is an alternate interior angle to the acute angle with vertex B on line *b*.)

$m\angle ACD + m\angle DCB = m\angle ACB = 30^\circ + 40^\circ = 70^\circ$

22. The two paths are the same length. Let the distance from point A to point D be presented by *t*. The length of the first path (half the circumference of a circle with diameter *d*) $t = 1/2\pi d$. The length of the second path (three halves of circles, one with diameter *a*, one with diameter *b*, and one with diameter *c*): $1/2(\pi a) + 1/2(\pi b) + 1/2(\pi c) = 1/2\pi\,(a + b + c) = 1/2\pi d$.[1]

23. The square peg in the round hole leaves relatively more open space. Let the peg have a radius of *r* in both cases.

Round peg in square hole:
Area of the square hole: $(2r)^2 = 4r^2$
Area of the round peg: πr^2.
Ratio of the open space to the total area of the hole:
$(4r^2 - \pi r^2)/4r^2 = 1 - \pi/4$, slightly less than 1/4.

Square peg in round hole:
Area of square peg: $(r\sqrt{2})^2 = 2r^2$
Area of round hole: πr^2
Ratio of the open space to the total area of the hole:
$(\pi r^2 - 2r^2)/\pi r2 = 1 - 2/\pi$, slightly more than 1/3.

24. The sum of the measures of the angles is 180º. Five triangles can be created in the star, each with two vertices at star points and one vertex as the interior angle of the convex pentagon with vertices abcde. The sum of the angles in these five triangles is $2(m\angle A + m\angle B + m\angle C + m\angle D + m\angle E) + (m\angle a + m\angle b + m\angle c + m\angle d + m\angle e) = 5*180^\circ = 900^\circ$. Notice that each of the vertices at the star's points is counted twice in this process and the 540º of the pentagon's angles (3*180º) are also included: Solving for the sum of the star's angles only, $(m\angle A + m\angle B + m\angle C + m\angle D + m\angle E)$, yields180º.

1 The function of auxiliary lines is to change difficult probelms to simpler ones, often ones which have already been solved. Auxiliary lines could also be drawn perpendicular to line a through point A, creating a quadrilateral whose angles include ∠ACB and can be solved, or perpendicular to line *c* through point C, creating two triangles, the solution of whose angles resolves the measurement of ∠ACB. An auxiliary line can also be drawn through points B and C; its intersection with line *a* creates a triangle, the solution of whose angles resolves the measurement of ∠ACB.

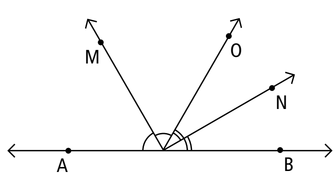
25. The person is 66 feet distant. Setting up a proportion from similar triangles: $2/22 = 1/11 = 6/x$, where x represents the distance to the person, in feet; $x = 66$ feet.

26. a. There are two symmetries that are reflections in lines, each reflecting through one of the two lines joining opposite vertices and there are two symmetries that are reflections in lines through the perpendicular bisectors of two opposite sides. There are four symmetries that are rotations: through 90º, 180º, 270º, and 360º (the last of which is the same as not rotating at all).

b. The symmetry transformation that results is a rotation through 90º either counterclockwise or clockwise. Note that a rotation of 90º in one direction can be thought of as a rotation of 270º in the other direction.

27. The parallelogram has a base of 1 unit, a height of 1 unit, and two sides that are not bases with lengths of 13/5 units. It can be drawn by drawing two parallel lines one unit apart, fixing a segment of unit length on one of the lines to be the base, then drawing a circle of radius 13/5 around one of the endpoints of the segment, and finding the intersection with the other line. The fourth vertex is then one unit farther along the second line. Alternatively, by the Pythagorean theorem, the length of the projection of the oblique side on the base line will be 12/5 units; so one could lay out a segment of length 12/5 = 2.4 on the line containing the base, then drawing the perpendicular and finding its intersection with the parallel line at distance 1.

28.



Angle MON measures 90º.
$2m\angle MOC + 2m\angle CON = 180^{\circ}$
Dividing all terms by 2 yields $m\angle MOC + m\angle CON = 90^{\circ}$
$m\angle MOC + m\angle CON = m\angle MON = 90^{\circ}$

29. It will hold 16 quarts of water (8 times as much!).
Volume of smaller container: $2 = V_1 = h\pi r^2$
Volume of larger container: $V_2 = 2h\pi(2r)^2 = 8h\pi r^2$
$V_2 = 8V_1 = 16$

30. A sum of 7 is more likely. There are five ways of rolling a 6: (1,5), (2, 4), (3, 3), (4, 2), (5,1) and six ways of rolling a 7, (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1). Hence the probability of rolling a 6 is 5/36 and the probability of rolling a 7 is 6/36 = 1/6.

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President, Houston Endowment Inc.
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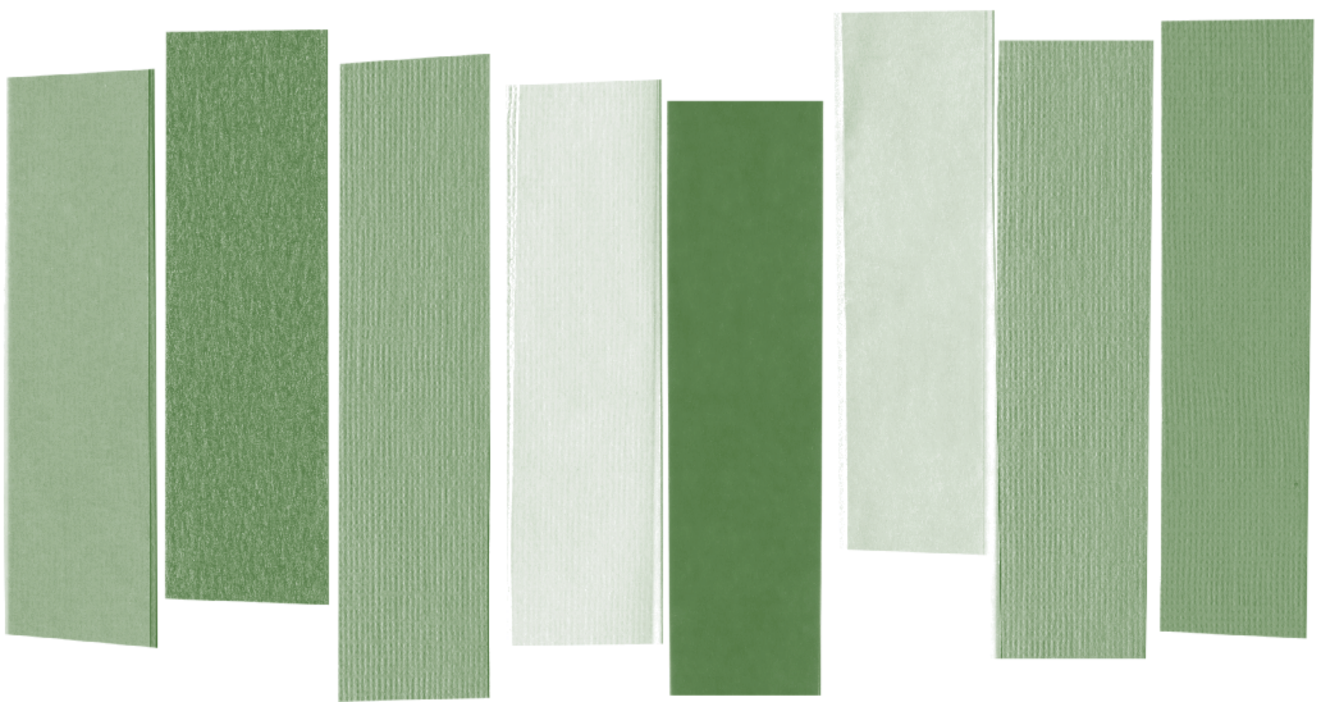

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National Council on Teacher Quality
1341 G Street NW, Suite 720
Washington, D.C. 20005
Tel 202 393-0020 Fax 202 393-0095 www.nctq.org

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