

Typical of many math textbooks in the U.S., this one is thick, multicolored, and full of games, puzzles, and activities, to help teachers pass the time, but rarely challenge students.

Singapore Math's textbook is thin, and contains only mathematics—no games. Students are given brief explanations, then confronted with problems which become more complex as the unit progresses.



Section 5
Dividing Decimals, Estimation, and Mental Math

IN THIS SECTION
 • Exploration 1
 • Exploring a Decimal
 • Estimation and Mental Math

The Perfect Pet

Selling the Slugs
 Many famous people have had unusual animals as pets. Take the Pet Quiz below and try to match the pet to the famous person.

Quiz

1. Adolf Hitler Emperor of Rome	2. Abraham Lincoln
3. Jacques de Basquirol, Emperor of France	4. George Washington
5. Henry II King of England	

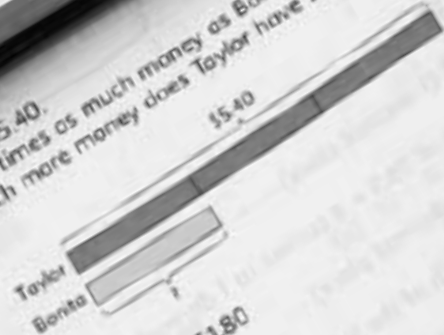
Match the Pet

A. warbler	B. giraffe
C. white bear	D. turkey
E. pig	

- Think About It**
- How many of the five famous people did you match with the correct pet (answers are at bottom of page)? Write your answer as a percent.
 - There were about 5.1 million pet dogs in the United States in 1991 and 3.5 million households owned at least one dog. Use a calculator to find the mean number of dogs in a household. Round your answer to the nearest tenth.
- Some of the most popular pets to own are dogs, cats, mice, hamsters, rabbits, snakes, lizards, fish, or birds.

Section 5 Dividing Decimals, Estimation, and Mental Math

25. Taylor has \$5.40.
 She has 3 times as much money as Bonita.
 How much more money does Taylor have than Bonita?



$\$5.40 \div 3 = \1.80

Bonita has \$1.80.

Taylor has \$ more than Bonita.
 60

Miracle Math

BY BARRY GARELICK

It was another body blow to education.

In December of 2004, media outlets across the country were abuzz with news of the just-released results of the latest Trends in International Mathematics and Science Study (TIMSS) tests. Once again despite highly publicized efforts to reform American math education (some might say *because* of the reform efforts) over the past two decades, the United States did little better than average (see Figure 1). Headquartered at the International Study Center at Boston College and taken by tens of thousands of students in more than three dozen countries, TIMSS has become a respected standard of international academic achievement. And in three consecutive TIMSS test rounds (in 1995, 1999, and 2003), 4th- and 8th-grade students in the former British trading colony of Singapore beat all contenders, including math powerhouses Japan and Taiwan. United States 8th graders did not even make the top ten in the 2003 round; they ranked 16th. Worse, scores for American students were, as one Department of Education study put it, “among the lowest of all industrialized countries.”

A successful program from Singapore tests the limits of school reform in the suburbs

Many professional mathematicians, concerned with the decline of math education in the United States, took a hard look at the Singapore Math methods and texts and liked what they saw.

During the clamor over the TIMSS results (released in December 2004), I heard Robyn Silbey, a math “content coach” from a Rockville, Maryland, public school, being interviewed by Ira Flatow for his *Science Friday* program on National Public Radio. Silbey worked at College Gardens Elementary. She explained that her school was one of four in the Montgomery County Public Schools (MCPS) district experimenting with Singapore’s math program. And, according to Silbey, it was working. The Singapore texts and methods were so effective in College Gardens that the scores of students there on the math computation portion of the standardized Comprehensive Tests of Basic Skills (CTBS) rose from the 50th and 60th percentiles to the low 90s in the first 4 years they were used.

I later learned that an evaluation of the pilot program conducted by MCPS found that in the schools where Singapore Math (SM) was being used as a pilot program, students typically outperformed their peers in other district schools. Yet despite these positive results, three of the four pilot schools dropped out of the program after fewer than four years. Why, I wondered. If the county’s own evaluation found benefits from Singapore Math, why not continue using it? In view of America’s disappointing rankings in math and Singapore’s record of success, why wasn’t the Singapore Math program given a serious and extended try?

In the Beginning

While the story of Singapore Math in Montgomery County does not answer all the questions about the persistently poor math literacy of American students, the failure of the program to take hold there does provide disturbing clues

about some of the institutional and governmental practices that impede improvement in education—and not just in Montgomery County.

In my early research into what happened in Montgomery County, I met John Hoven, then co-president of the Gifted and Talented Association of Montgomery County and now a national advisor to NYC HOLD (New York City Honest Open Logical Decisions on Mathematics Education Reform), a non-partisan advocacy organization that provides information to parents, teachers, and others on math education issues. Hoven, an economist in the Antitrust Division of the U.S. Department of Justice by day, had discovered Singapore Math while waging a successful battle to get MCPS to forgo a National Science

Foundation (NSF) grant worth \$6 million. The grant would have trained teachers to use a middle-school program called Connected Math, one of several, Hoven learned, that was funded by the Education and Human Resources Division of the NSF and based on standards developed by the National Council of Teachers of Mathematics (NCTM). Though the NCTM is a private organization, it exerts enormous influence over the math standards and texts used by most states and districts in the United States—standards and texts that, in Hoven’s view, were failing.

During the campaign against the Connected Math grant, Hoven discovered Singapore Math. He learned that Singapore, whose population is half that of New York City, had begun modifying its education policies in the early 1980s to build up its labor force in such a way as to create technical skills unavailable elsewhere in the Third World. The Curriculum Development Institute of Singapore (now called the Curriculum Planning & Development Institute) had created the math program and the accompanying texts, called *Primary Mathematics* (which were published in English, Singapore’s official administrative language, in 1982), to help boost that technological prowess. The *Primary Mathematics* series was at the heart of Singapore’s national math curriculum as it achieved its successive TIMSS victories.

Many professional mathematicians, concerned with the decline of math education in the United States, took a hard look at the Singapore Math methods and texts and liked what they saw. The texts have been distributed in the United States by an Oregon company since 1998 and are used by many homeschool parents and promoted by Internet-based parent and professional organizations. In addition, the private nonprofit Rosenbaum Foundation helped fund the implementation of

Singapore Math programs in scattered sites around the United States and in Israel. A study of Singapore Math conducted by the American Institutes for Research (AIR) for the U.S. Department of Education (released in January 2005) concluded, “What the United States needs overall are the sound features of the Singapore Mathematics system.” In studying several different American school districts that were experimenting with the program, including the Montgomery County Public Schools, AIR researchers found that “Singapore Mathematics textbooks can produce significant boosts in achievement.” But the AIR report also cautioned that making Singapore Math work in the United States “will require the same sustained commitment to developing a quality mathematics system that Singapore gave to its reform efforts.”

That was a lesson still to be learned in Maryland.

A Long Way from Singapore to Montgomery County

John Hoven and his allies persuaded the Montgomery County Public Schools to try Singapore Math instead of pursuing the Connected Math grant, but instead of receiving \$6 million, the district would have to spend its own funds. And that wouldn't be \$6 million. “The initial plan was for a \$50,000 pilot spread out over two years,” Hoven recalls. “It was \$50,000 in a \$1 billion budget. The money would pay for textbooks and nothing else—no teacher training, nothing.” Hoven knew it was just a drop in the bucket for the district, but he was sure that the Asian math program would sell itself.

MCPS selected four middle-class, ethnically diverse, suburban schools—College Gardens in Rockville, Charles R. Drew and Highland View in Silver Spring, and Woodfield in Gaithersburg—to participate in the pilot. But few teachers at the schools realized how different Singapore's approach to math was from what they had been used to; it was nothing less than a total shock to the schools' systems.

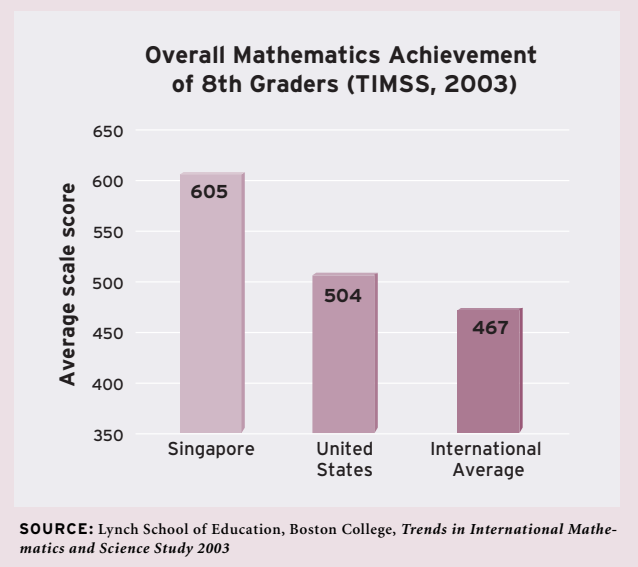
Unlike many American math textbooks, such as *Math Thematics*, published by Houghton Mifflin, which are thick, multicolored, and multicultural, Singapore's books are thin and contain only mathematics. There are no graphics (other than occasional cartoons pertaining to the lesson at hand), no spreadsheet problems, and no problems asking students to use a calculator to find the mean number of dogs in a U.S. household. With SM, students are required to show their mathematical work, not explain in essays how they did the problems or how they felt about them. While a single lesson in a U.S. textbook might span two pages and take one class period to go through, a lesson in a Singapore textbook might use five to ten pages and take several days to complete. The Singapore texts contain no narrative explanation of how a procedure or concept works; instead, there are problems and questions accompanied by pictures that provide hints about what is going on.

According to the AIR report, the Singapore program “provides rich problem sets that give students many and varied opportunities to apply the concepts they have learned.”

Another key difference is the number of topics covered by Singapore's texts for a single grade. The AIR study frequently criticizes American math texts for being an inch deep and a mile wide, covering a great range of topics with little time spent on developing the material, including mastery of math facts. (One of the texts with which the AIR study compares Singapore's Primary Mathematics series is *Everyday Mathematics*, a program developed with NSF funding and used widely in Montgomery County.) The MCPS 1st-grade curriculum goals, for instance, contain a number of nonessential topics, such as sorting concrete objects (like Post-its with names of favorite pets on them) into categories, activities that take up instructional time which, critics of the MCPS curriculum argue, could be better

A Model Country (Figure 1)

Singapore's 8th graders had the highest achievement on a 2003 international math test, outscoring American students by a full standard deviation, equivalent to about three to four grade levels of schooling.



spent laying the foundation for algebra in 8th grade.

Singapore's texts also present material in a logical sequence throughout the grades and expect mastery of the material before the move to the next level. In contrast, mainstream American math texts and curricula frequently rely on a “spiral” approach, in which topics are revisited and reviewed. The expectation of that approach is that not all students achieve mastery the first time around. One Ohio school teacher familiar with the spiral approach summed up much of the criticism of

Bar None *A Secret to Singapore's Success*

A typical problem given to math students in the lower grades goes like this:

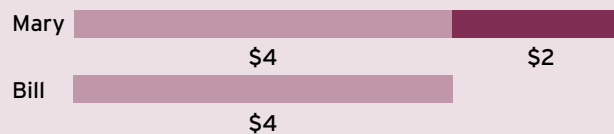
“Mary and Bill have \$10 between them. Mary has \$2 more than Bill. How much money does each person have?”

In the United States, students are expected to solve such problems using an approach called “Guess-and-Check,” a trial-and-error approach in which the children “try out” combinations of numbers and check to see if they get the right answer. In Singapore, students are taught to solve such problems using the bar-model method. Bar modeling offers an arithmetic and exact approach to math challenges. No guessing. In the problem given, for example, students would draw two bars, labeling one Mary and the other Bill. The sum of the “bars” is \$10. Mary’s bar is a little longer than Bill’s, indicating that Mary has \$2 more than Bill.



The extra \$2 portion of Mary’s bar is darkened so that the undarkened portion of Mary’s bar is equal in length to Bill’s. Removing the \$2 results in two bars of equal length. Now the sum of the two bars is \$8 (since \$2 has been subtracted from the total), leaving the student to solve the problem of 2 times the short bar (call it B for Bill) = \$8.

The student has learned how to solve this problem and divides by 2 to obtain \$4 for Bill. Adding back in the \$2 for Mary now gives the solution: \$4 + \$6 = \$10, and the difference is clearly \$2.



With practice, Singapore Math students learn a simple technique for solving a variety of math problems. The result, as the international tests seem to show, places them ahead of their Guess-and-Check American contemporaries.

— Barry Garelick

the method on an Internet math forum, saying, students “can’t remember how to do it when [they] do return—or if they do remember it, it’s now being taught in a different way.”

The most important feature of Singapore’s texts is an ingenious problem-solving strategy built into the curriculum. Word problems are for most students the most difficult part of any mathematics course. Singapore’s texts help students tackle them through a technique called “bar modeling,” in which students draw a diagram to help them solve the problem. Typically, in U.S. texts, students are taught to use a method called “Guess and Check”—trying combinations of numbers until the right numbers are found that satisfy the conditions of the problem—a method that many professional mathematicians consider inefficient (see sidebar). The bar-modeling technique not only provides a powerful method for solving problems, but also serves as a link to algebra. Symbolic representation of problems, the mainstay of algebra, emerges as a logical extension of the bar-modeling technique.

What Happened in Montgomery County?

Given all of the mathematical strengths of the Singapore program, why was the pilot abandoned so quickly in Montgomery County? The simplest answer is that where Singapore

Math worked the best, in College Gardens, it is still being used; where it didn’t work as well, it was dropped. But that does not begin to explain what happened.

All four Montgomery County schools used the Singapore Math texts in 2000–01 and 2001–02, but only College Gardens and Highland View kept the program in 2002–03. The “math computation” scores at College Gardens show a dramatic improvement for both 2nd and 4th grades (see Figures 2a and 2b), but in “general math” there is no discernible pattern; all four schools had either no change or a decrease in scores.

Additional results from the pilot were detailed in the evaluation conducted by the MCPS Office of Shared Accountability after the second year of the experiment. The county evaluators found that students in the four Singapore Math pilot schools generally progressed through the curriculum at an accelerated pace compared with their peers in control schools. But while the school district’s evaluation was positive in tone (Singapore Math “helped prepare students for higher-level math placements in middle school”), it reported mixed results and offered no recommendation for expansion.

Because the effectiveness of a program as sophisticated and multidimensional as Singapore Math cannot be thoroughly evaluated in just two years of testing, the story of its failure in

Montgomery County says more about school politics and finances than about math programs. (It would help, for instance, to track students who went through Singapore’s program through their 8th-grade tests to ascertain how well they were prepared for algebra.) The mixed math results of the county’s evaluation should have been seen for what they were: an interim assessment. Instead, the county ended the funding for the program after the second year. If schools wished to continue, they had to pay for the materials out of their own budgets, which they didn’t need to do if they used district-approved texts such as *Everyday Mathematics*.

Detailing the many reasons for dropping support for the pilot by Montgomery County without waiting for long-term results would take more space than is available here. But we can get a sense of the thing by examining some of the reasons that the three schools gave for quitting Singapore Math and those given by College Gardens for staying with it.

The first problem was lack of planning and preparation. The depth and breadth of the differences between Singapore Math and American math were not appreciated. The decision to use Singapore Math was made in 1999, for instance, but textbooks and other teaching materials did not arrive at the four schools until late spring the following year, giving teachers just three months to prepare to introduce the program to students in the fall of 2000.

The Singapore Math manuals were another problem: they provided very little guidance on how to teach a particular lesson—because they are written for teachers who, for the most part, have a deeper understanding of mathematics than most U.S. teachers do. That dilemma was compounded by the lack of experience with Singapore’s program by Montgomery County and its delay in training teachers to use it. The Montgomery County Public Schools eventually developed a training program, but some people believe it was too little, too late. College Gardens and Highland View found funds to hire Singapore Math specialists (like Robyn Silbey) to help get the program off the ground and coordinate the training within their schools. Scott Baldrige, a Louisiana State University mathematician who provides professional training to teachers in implementing the Singapore program, believes that such training helps. “Some teachers get it on their own,” he says, “but many need professional development to see how the curriculum interacts with the students over several years.”

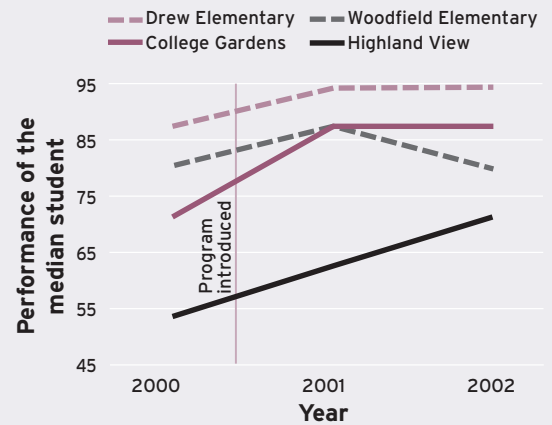
Even with adequate training, the two-year span of the pilot resulted in three of the pilot schools (all except College Gardens) introducing Singapore Math all at once, across all grades, which put older children at a severe disadvantage, since Singapore Math concepts build on one another. This helps to explain the difference in the math test results.

Another complaint expressed by teachers and administrators in all four schools was that Singapore Math was not in line with state standards. Indeed, the state’s academic standards

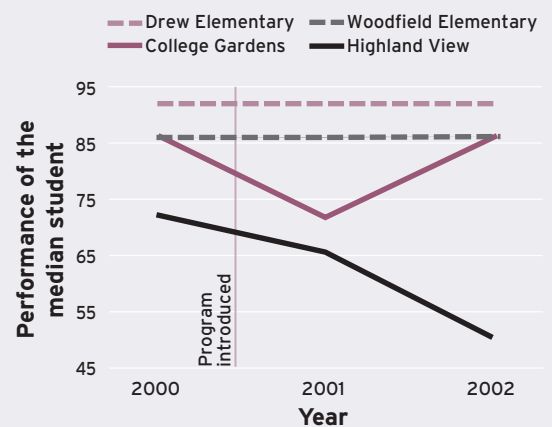
Early Evidence (Figures 2a and 2b)

Two years after Singapore Math was introduced, 4th graders in the four Montgomery County (Maryland) schools implementing the program scored higher in math computation. However, the scores on the general mathematics section were flat or negative.

Math Computation Performance of 4th Graders Before and After the Introduction of Singapore Math



General Mathematics Performance of 4th Graders Before and After the Introduction of Singapore Math



Note: Performance is the national percentile rank on the Comprehensive Tests of Basic Skills (CTBS).

SOURCE: Maryland State Department of Education

include data analysis, statistics, and probability, which Singapore’s texts do not address. But it is more complicated than that since, by state law, each school district has authority over its own curriculum. Although required to administer the state tests, school districts are not required to align their

Rather than waste students' time with inefficient methods for solving problems, Singapore's texts provide instruction that eliminates trial and error, one of the goals of mathematics.

curriculum with the state standards. That means, says Hoven, that “they can choose to aim for world-class standards instead, which at one time was one of the goals of the MCPS Long-Range Plan.” But the plan was revised to be aligned with state standards instead—a move, say some, that was, in effect, a decision to lower Montgomery County’s standards.

This brings us to the question of money. Eileen MacFarlane, principal at Drew, said that her school initially supplemented Singapore Math with additional material on statistics, data, and probability to cover the misalignment, but the cost of purchasing Singapore’s materials from their own budget became a problem.

Joanne Steckler, now retired as principal of Highland View, said, “No one told us to drop the program.” But being required to buy its own materials had the same effect. “For one year we did purchase our own Singapore materials,” said Steckler. “But we did not want other curriculum areas to suffer because of lack of funds to purchase materials, so we gave up Singapore Math.”

Shawn Miller, principal of Woodfield Elementary, also cited the cost of the texts. Once the county stopped funding Singapore Math, Miller made the decision to go with the county-approved program, Everyday Mathematics, which, he said, “was better aligned with the state curriculum.”

With such standards-alignment and budget concerns, the schools had a safe way to bow out of the pilot.

A Culture Shock to the System

But budget questions were hardly fundamental. Taking on a program like Singapore Math meant going against what many teachers believed math education to be about; surely,

it was not what they were trained for. Since the success of Singapore’s programs relies in many ways on more traditional approaches to math education, such as explicit instruction and giving students many problems to solve, in some ways its very success represented a slap in the face to American math reformers, many of whom have worked hard to eliminate such techniques from the teaching canon.

Gail Burrill, a former president of the NCTM, suggests quite bluntly that the success of Singapore Math cannot be imported. “These are books used by a different culture, a culture that is more homogeneous, and a culture that has a consistent way of thinking about mathematics.” And Cathy Seeley, a former president of the NCTM, hints as

much by arguing that Singapore’s success (as well as that of other Asian countries) is not about the textbook. “We have to look beyond their textbooks to determine what these lessons are.”

The logic of the argument that it is the Asian culture or something “beyond their textbooks” that produces math success leads to the conclusion that, as NCTM adherents often contend, content doesn’t matter nearly as much as the teacher or the culture that produces the “proper conditions for learning.” Eileen MacFarlane maintained that the teachers in the Singapore pilots drew from the texts, but then quickly added, “The text is a resource, not a curriculum.” She said this despite her enthusiasm for Singapore’s program.

But the belief that the difference between Singapore Math and American math is just in the teaching or, as some suggest, the culture, is a rationalization, says David Klein, a mathematician at California State University, Northridge. “Math reformers assume that math education is bad in the United States because the NCTM reforms were not properly implemented nor understood by teachers,” he continues. “They never consider the possibility that the NCTM standards themselves and the textbooks written for those standards are one of the causes of poor math education in this country.”

The only person I heard openly disagree with the “teacher not text” argument was Dr. Sherry Liebes, then the principal of College Gardens, the only school that kept the program. While she said that teacher training is important, she added that Singapore’s texts provide a structured curriculum, and thus “It’s one less thing for teachers to worry about.” This notion was echoed in the AIR study, which quoted a teacher in one of the pilot schools in Montgomery County: “Having to explain Singapore Mathematics made

me understand that I never really understood the mathematics I was teaching.”

Another stumbling block for the Maryland teachers was their concern that the Singapore Math program did not contain “real-world” activities. The term, as used by those who follow the ideas supported by the National Council of Teachers of Mathematics and education schools for teaching math, generally means a problem for which American students have not received much instruction or preparation. This is intentional, it turns out, because it is believed to be good for students to learn to approach problems for which they have not received explicit preparation. The National Education Association (NEA), for instance, in its online version of “A Parent’s Guide to Helping Your Child with Today’s Math,” gives an example of a “real-world” problem:

A farmer sends his daughter and son out into the barnyard to count the number of chickens and pigs. When they return the son says that he counted 200 legs but the daughter says she counted 70 heads. How many pigs and chickens does the farmer have?

The NEA then suggests that some students may solve the problem using algebra (those who know how to do so, that is), while others might solve it using Guess and Check. Still others may choose to draw pictures to solve it. The NEA admits that some methods might be considered more efficient, but points out that the correct answer can be found using multiple methods and that “by allowing students to think flexibly about numbers, we encourage them to ‘own’ the math forever, instead of ‘borrowing’ until class is over.” That this real-world problem depicts an approach that no sensible person would use in counting pigs and chickens is beside the point.

This kind of real-world math is indeed missing from Singapore’s program—apparently, if TIMSS tests mean anything, without much harm. Rather than waste students’ time with inefficient methods for solving problems, Singapore’s texts provide instruction that eliminates trial and error, one of the goals of mathematics. Bar modeling is a powerful pictorial technique that results in one answer, deduced by using mathematical principles that students have learned rather than by employing the haphazard trial-and-error method of Guess and Check.

For One Brief Shining Moment

An exact description of which differences in math instruction matter most is perhaps impossible. For instance, an emphasis on sequential mastery of skills that builds on previously acquired skills is a key component of the Singapore Math program and not important in the American approach, where activities don’t require such skills. While the latter creates the

illusion of equal achievement, international tests like the TIMSS would seem to provide a reality check on that illusion.

The struggle to make math instruction work, of course, is not limited to Montgomery County. In the state of Washington, parent protests against the adoption of several standard math curricula (like Connected Math) led a state representative to introduce a bill earlier this year to put Singapore Math in all the state’s elementary schools. And in New York City, Elizabeth Carson, who heads NYC HOLD, has led a battle for years to rid the city’s schools of programs like Everyday Mathematics. She calls it a “tragedy for our children and our nation” that American attempts at math reform “bear no resemblance to the programs and standards of the highest-achieving nations.”

Having watched as three of the four schools dropped Singapore’s program in Montgomery County, John Hoven shared Carson’s concern. Discouraged, he resigned a year ago from the county’s Gifted and Talented Association. “I had stopped believing I could make a difference,” he says. “I felt it was time for someone else to try.”

In the meantime, the decline in the numbers of U.S.-trained scientists and engineers, compared with the increasing numbers of those trained in Asian countries, has not gone unnoticed. In this year’s State of the Union address, President George W. Bush stated: “We need to encourage children to take more math and science, and make sure those courses are rigorous enough to compete with other nations.” He proposed “to train 70,000 high-school teachers to lead Advanced Placement courses in math and science . . . bring 30,000 math and science professionals to teach in classrooms . . . and give early help to students who struggle with math, so they have a better chance at good, high-wage jobs.”

A few months later, President Bush created the National Mathematics Advisory Panel to advise the White House and the secretary of education on the best use of scientifically based research to advance the teaching and learning of mathematics. The panel includes several people who have actively fought against the NCTM-led “fuzzy math” trend in this country.

While the goal of bolstering high-school math is a laudable one, the success of high-school students in math depends on what they’ve learned in the lower grades. If those foundations are weak, the addition of Advanced Placement courses in math and science in high schools will prove to be a weak enhancement. Unfortunately, changing the way math is taught in the lower grades appears to threaten an education philosophy and method that is pervasive in our schools, and does not move us towards academic excellence.

Barry Garelick is an analyst for the federal government and lives in the Washington, D.C., area. He is a national advisor to NYC HOLD, an education advocacy organization that addresses mathematics education in schools throughout the United States.